The **ScienceMath-**project: **Sound Functions**
Idea: Damjan Kobal,
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Teaching Material

Sound Functions

Digital technology has revolutionized electronic communications. A simple idea based on understanding of continuous and discrete functions can explain the essence of digital technology, which can 'squeeze ever more information on ever smaller bits of space'. In particular, it is easy to explain the idea of how dozens of telephone conversations can be squeezed into a single phone line.

A web version of the lesson can be found at
http://uc.fmf.uni-lj.si/com/Digimusic/digimusic.html.
We strongly recommend visiting it as some of the ideas only show its value via interactive response.

INTRODUCTION

Many mathematical ideas can be given a meaningful intuitive insight by sounds. For example, prescribing different tones to different digits we can 'listen' to many 'mathematical meanings'. For example a finite decimal number can be presented as a simple (finite) 'melody':

$$0.75$$

Such a finite decimal presents a rational number as does a fraction

$$124/999 = 0.124124124124124124.....$$

The repeating (periodic) 'melody' nicely describes the 'rational nature' of the rational number and the infinite decimal number is presented by infinite (though monotonous) melody. How could one better explain the difference with irrational number as to play 'the infinite and never repeating' melody of for example:

$$\sqrt{2} = 1{,}4142135623730950488016887242097\ldots$$

Or the well known

$$\pi = 3{,}14159265358979323846264338327 95\ldots$$

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In communication technology it is important that any sound, conversation, recording, ... can be presented as a function. We will not get into details of how a common sound is presented as a function. For example musicians know, that a perfectly sounding *A* can be described by a function Sin[Pi 440 t], or playing one octave higher is a function:

$$\sin(2\pi\ 440\ t)$$

By means of functions we can easily get more sophisticated 'sounding functions'.

$$\sin(34+\sin(950t))$$

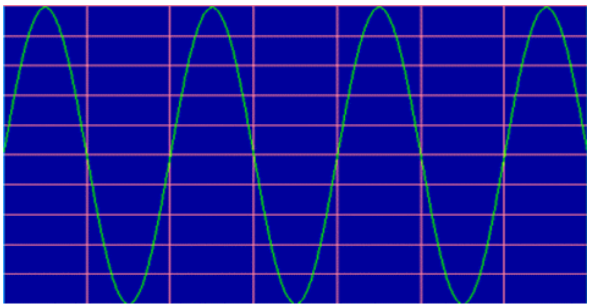
$$\sin(700t+35t\cdot\sin(123t))$$

$$\sin(700t+\cos(150t)+45t\cdot\sin(350t))$$

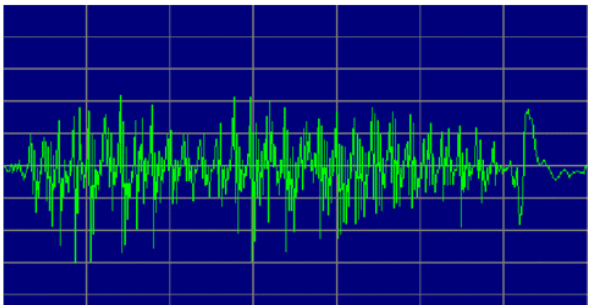
The above functions can be ‘played and listened’ on the web version of the lesson. The sounds were produced by program *Mathematica*, where one has a command Play, which is very similar to a command *Draw*.

DIGITAL TECHNOLOGY AND DISCRETE FUNCTIONS

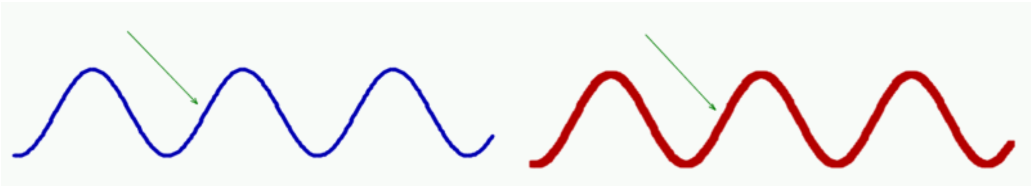
Functions that describe 'artificial sounds' or pure tones (of a single frequency) are of 'orderly shapes' like this pure tone described by a sin function (similar to the above *A* tone function):



A simple human voice 'hello' is a much more complicated function like the one seen in the bellow graph. Of course, how a function looks like depends very much on the perspective, that is how close a look we take...

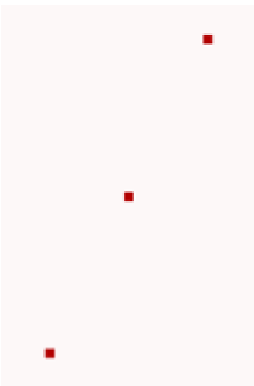


But whatever the sound, imagine now, that the sound is presented by a function. A true shape of a sound function is not essential for our ideas. Thus, let us say that we could basically take any function to present a sound. Different functions would present different sounds. Let us start with a simple *sin* function:



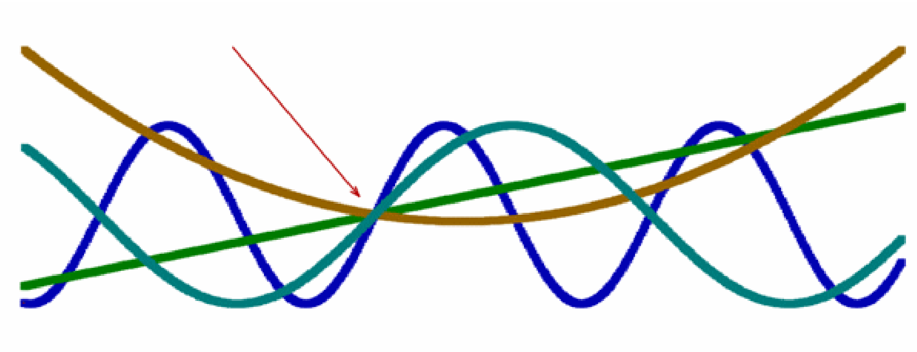
We drew two functions, which look very much the same. Imagine that the blue function is a sound that is recorded on one side of the phone line and the thicker, red function is a reproduced sound on the other side of the phone line. Functions look exactly the same and it seems just to say, that phone line service provider is doing a good job, transmitting a perfect copy of the sound from one side to the other. But let us take literally a closer look and let us focus on both graphs at the point indicated by the green arrow:

Phone provider can 'cheat' and only transmit a discrete function, which consists of points at a certain distance. Of course, points have to be dense enough for customers, talking on the phone, not to notice any 'empty spaces'. Certainly, if the provider would only 'transmit' a point every five minutes, we could hear nothing. But if one imagines a point every millionth of a second, what we get is 'very smooth' looking function. Our above graph is a graph of a same sin function. The blue is an 'analogue' continuous function, while the red one only has about 150 points for one period of a sin. What would the phone provider gain with such a cheating? Well, whenever we draw a point, we draw it of a certain thickness, but point's true thickness is 0. Imagine, we take further focus to the above point of the red graph. What we get looks like this:

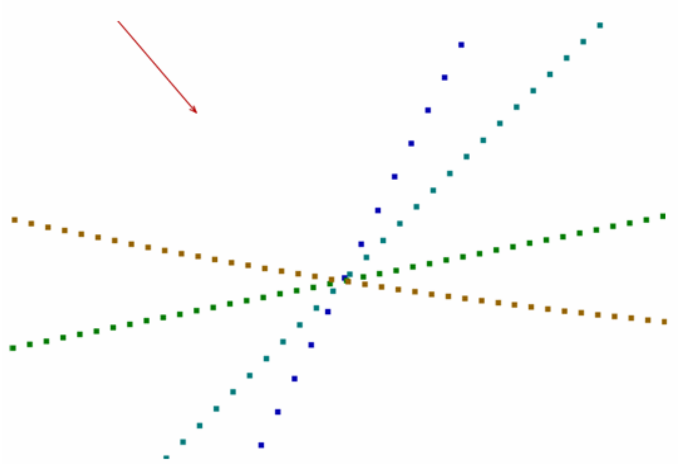


It is now pretty obvious, why the phone line provider would cheat like this. As the technology has long time ago won the race with human sensitivity and it can 'split the time' to far tinier bits than a human ear could notice, we see, that technology offers the provider 'lots of free time'. The necessary density of points is determined by human ear sensitivity, and if the technology offers the 'split of time' to tenth of interval that human ear can notice, the machine can be programmed to 'listen' only one tenth of a time and is free nine tenths of a time. Basically, we see, that if we imagine the above red 'dots' as discrete values of a function, we can squeeze ten other points in-between.

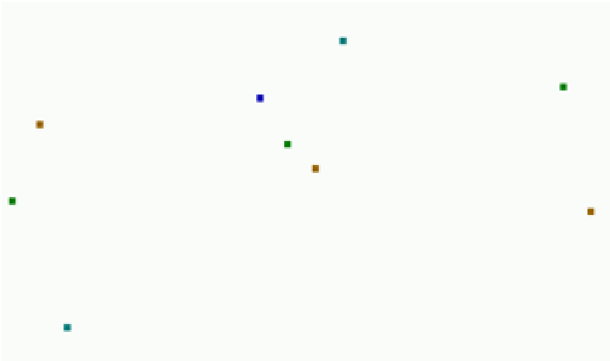
Let us look at the graph of four different functions. We made the functions to intersect (at a point indicated by red arrow) just to make it easier to explain our idea. It seems we have four different and 'precisely described functions'. It is clear that if we think of functions as sounds, this picture could easily be transformed (imagine colour filter) into four different (clearly heard) sounds. And this is basically the trick of digital technology. People talking on a phone and real life users and customers of audio technology have very limited ear sensitivity and can be fooled to truly hear four different sounds from the bellow picture. In fact we can say, that our eyes were fooled to see four different functions bellow, while mathematically (that is precisely) speaking, we even do not have one function defined at the whole visible interval.



To see and comprehend what we are talking about, let us again focus to the intersection point indicated by red arrow.



And an even closer focus reveals a truly different picture:

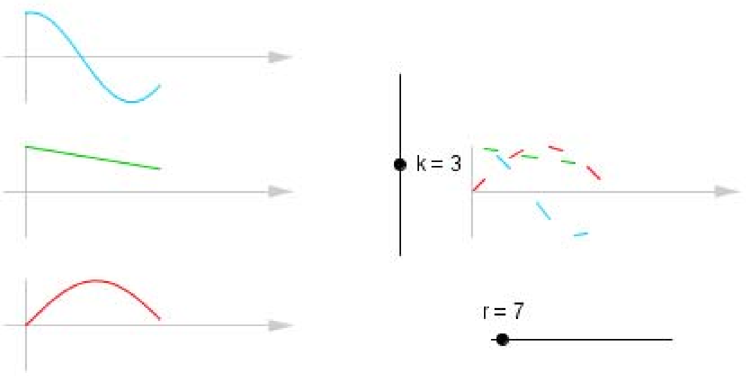


We see, that 'much less than one digital' function can be made to carry enough information to reproduce 'four different functions', that are still precise enough to carry all the information necessary for human ear to 'hear a continuous sound'.

Understanding the essence of functions and discretely defined functions, it is pretty obvious that this process looks unlimited. How many functions like that can be 'squeezed' into one discrete function? How many phone conversation can be squeezed into a single phone line? Of course it depends on the quality of the sound required (density of discrete points, which represent particular functions) and on the ability of technology to 'listen' and 'record' ever shorter bits of time.

In reality, the machine would not only record bits of conversation on evenly spread out intervals, namely, that could result in recording noise (sound pollution), but the machine would record all the conversation and transmit only (noise filtered) average for those tiny bits of time.

On the web version of the lesson one can observe 'the recorded' sounds on the left and the transmitted sounds on the right. The web applet nicely simulates the transmission of a sound. On the left we see the three channels (three different sounds), in the middle we see how much information the machine picks up of each channel and on the right are 'rebuild' functions (sounds). By changing the resolution (r), that is the density, the sensitivity of a machine, we can reproduce quite authentic sounds on the right, even if we increase the number of channels (k) in a single phone line (function).



With modern computer technology this wonderful and simple idea can easily be simulated by dynamic presentation of functions, when 'zoom in' and zoom out' can nicely and intuitively visualize how relative to human eye and ear a discrete or continuous looking functions can be.

Finally, not as a complete joke, the idea can be given a funny but meaningful parallel. Imagine a class of students taking a test and a teacher taking care that the students would not cheat. If a teacher leaves the classroom unattended, students might be tempted to start communicating and cheating. So it is hard to imagine, how the same teacher could take care of two different classes of students in two different classrooms at the same time. But that is because a teacher would be forced to leave at least one class unattended. But for how long? Imagine the teacher's strict eye is searching around the classroom every second... Theoretically, imagine the teacher who could shift its full presence and attention from one class to the other in tenth of a second. Is it not obvious that in such circumstances one such a speedy teacher could attend not only two but ten classes simultaneously?

Interactive simulation

Interactive computer simulations which greatly enhance the ideas can be viewed and listened on the web.

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