



Background

General didactical background

The basis of this sequence is an interdisciplinary approach with sciences especially with Physics. The pupils shall experience Mathematics in an appropriate, interesting and important way by the means of extra-mathematical references. Learning in interrelations shall contribute to an intuitive mathematic understanding. With the aid of scientific contexts and methods the gap between formal mathematics and authentic experience shall be closed and on the other hand the variety of mathematic items shall be experienced.

Scientific contents open a chance for realistic teaching. Concrete physical or biological phenomena can stimulate modelling processes and can lead to authentic experiences. Mathematical themes and methods are learnt in meaningful contexts; reality can be expanded by adding a mathematical view. Different real contexts lead to different models and can show various meanings of concepts and models. The richness of scientific phenomena allows open tasks and hands-on approaches to mathematics. Mathematical concepts, like the concept of variable, can be experienced as a modelling tool. In various authentic contexts the multifaceted meanings can be explored.

Mathematics didactical background

Malle (1986) differentiates three aspects of the concept of variable. *Variable as an object* stands for an unknown item or an unknown object. Variables referring to the *Placeholder aspect* are seen as placeholders, which you can substitute through a number. Variables which stand for a meaningless symbol, with which you can apply certain rules, belong to the *Calculational aspect*. He further differentiates into static and dynamic components. Wiegand and Jordan (2005) identify each aspect with different kinds of applying variables. Calculational aspect refers to technical tasks, like applying rules. Placeholder aspect refers to arithmetic tasks and variable as an object to conceptual tasks. Tasks designating to the calculational aspect afford the least cognitive complexity whereas tasks referring to variable as an object afford the most cognitive complexity. Malle demands to consider all aspects of the concept of variable, but most attention must be paid to variable as an object, especially in the beginning.

Trigueros et. al. (1996) have designed a 3x3 matrix which contains different representations of the concept of variable. They distinguish generalized number, representing a number in a continuous set of numbers and specific constant, representing a constant number, which might change in a different situation/context. A specific constant can also be identified as a representative of a discrete set of numbers. Further they include variable in a functional relationship. All representations can be seen at different levels: conceptualisation and symbolisation, interpretation and manipulation.

DECOMPOSITION OF VARIABLE

	Conceptualisation and Symbolisation	Interpretation	Manipulation
Generalised Number	Conceptualisation of a general object involved in general methods or rules deduced from numeric and/or geometric patterns and families of similar problems; and its symbolisation.	Interpretation of a symbol as a general object in algebraic expressions or in general methods.	Factorise, simplify and expand to rearrange expressions.
Specific Unknown	Conceptualisation of an unknown in a particular situation and/or in an equation and its symbolisation .	Interpretation of a symbol as a specific unknown in equations in which it appears once or more times.	Factorise, simplify, expand, transpose or balance an equation to make a variable the subject of equation.
Variable in a Functional Relationship	Conceptualisation or symbolisation of functional relationships starting from a table or graph or a problem in natural language.	Interpretation of correspondence and joint variation in analytical expressions, tables and graphs.	Factorise, simplify, expand to rearrange an expression, substitute values to determine intervals of variation, maximum/minimum values and global behaviour of the relationship.

Table 1: decomposition of variable according to Trigueros et.al. (1996)

The idea of teaching implementation

By using physical experiments students work with concrete measurands. These measurands shall be identified with variables. Hence variables will get a meaning. This makes it easier for students making sense of an abstract variable. That is in accordance to Malle, who demands to introduce the concept of variable by using variables as objects.

While doing the experiments, students experience that variables do not stand for a certain number, but represent a whole set of numbers. Besides that they discover the functional relationship between the two measurands in an authentic context. Both static and dynamic aspect of a functional relationship is touched. Especially the covariational aspect is touched, i.e. change of one variable causes a change of the other variable.

Students implicitly discover the different representations of a variable before they are theoretically introduced in class. Hence they will already have an imagination of that abstract object.

The main emphasis of this teaching sequence is on the concept of variable. But while working through the sequence all phases of the modelling cycle are touched. A scientific problem is examined and described by mathematical terms and reflected.

The **ScienceMath**-project: **Refraction and Concept of Variable**

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The instruction sheet starts with a problem from everyday life which shall link to students' experiences. After measuring values, they shall find a relationship between the two measurands (angle of incidence and angle of refraction). Once they found a formula, they shall discover the different aspects of the concept of variable.

After the experiments the formulas which the different groups have found can be compared in a classroom discussion. So students can realize, that the same objects can be represented by different variables, but still describe the same situation. Further equivalence of equations could be discussed and would serve as an introduction into this topic.

If there isn't enough material for a whole class, this experiment can be combined with thermal expansion, Boyle's law and buoyancy experiment.