The **ScienceMath** Project: **Proportions**
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Education and Culture
Lifelong learning programme
COMENIUS

Teaching Material

The working sheets can be used one after the other. They are prepared for self-dependent work. Aim is to deepen the knowledge about centric dilation/ similarty resp. proportions. Higher achievers can acquire this theme by the worksheets. Working sheets should be controlled by the teacher.

Hint: The working sheets are separated into two collections, which can be used combined or independent.

Working sheets of the first collection (1-5) concern the topics

- Centric dilation and similarity in geometry
- Relation between mass and volume (at same density)
- Similarity between “similar” animals like house cat and wild cat, adult animal and young,
- Allometries and biological perspective.

Working sheets of the second collection (6-9) concern the topics

- Relation between volume and surfice area
- Comparison of animal sizes and its reason from a mathematical and biological perspective: oxygen-, energy supply and the relation to proportions and behaviour.

Hint: For a self-dependent work of the students we recommend further access to background information (books, internet etc).

Working sheets

Collection I:

Working sheet 1: Centric dilation
Working sheet 2: Relation between mass and volume
Working sheet 3: Similarity
Working sheet 4: Is their similarity of animals?
Working sheet 5: Allometries

Collection II:

Working sheet 6: Are there bodies with same volume, but different surface area?
Working sheet 7: Locomotion behaviour and body forms of insects.
Working sheet 8: Oxigen supply of insects and endotherns and body size.
Working sheet 9: Heat transfer of endotherms and body size

Dieses Projekt wurde mit Unterstützung der Europäischen Kommission finanziert. Die Verantwortung für den Inhalt dieser Veröffentlichung (Mitteilung) trägt allein der Verfasser; die Kommission haftet nicht für die weitere Verwendung der darin enthaltenen Angaben.

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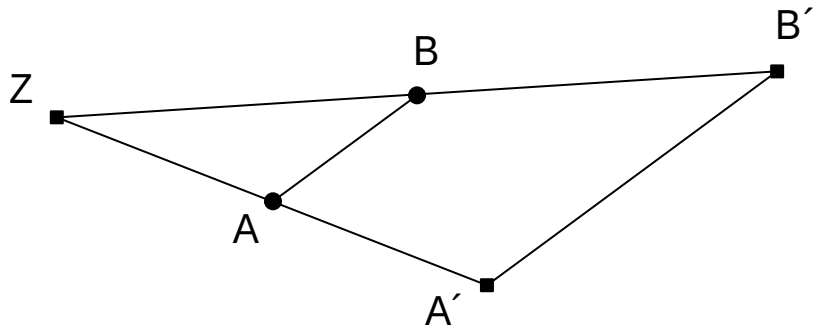
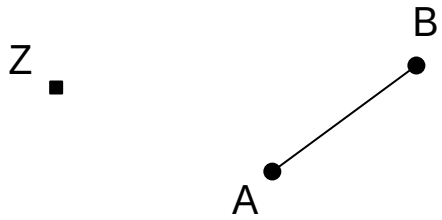
Working sheet 1: Centric Dilation



Distance AB is streched by streching factor k = 2 at centrum Z.

Task:
Strech AB by k = 3 at Z.



The figures at the following page result from centric dilation.

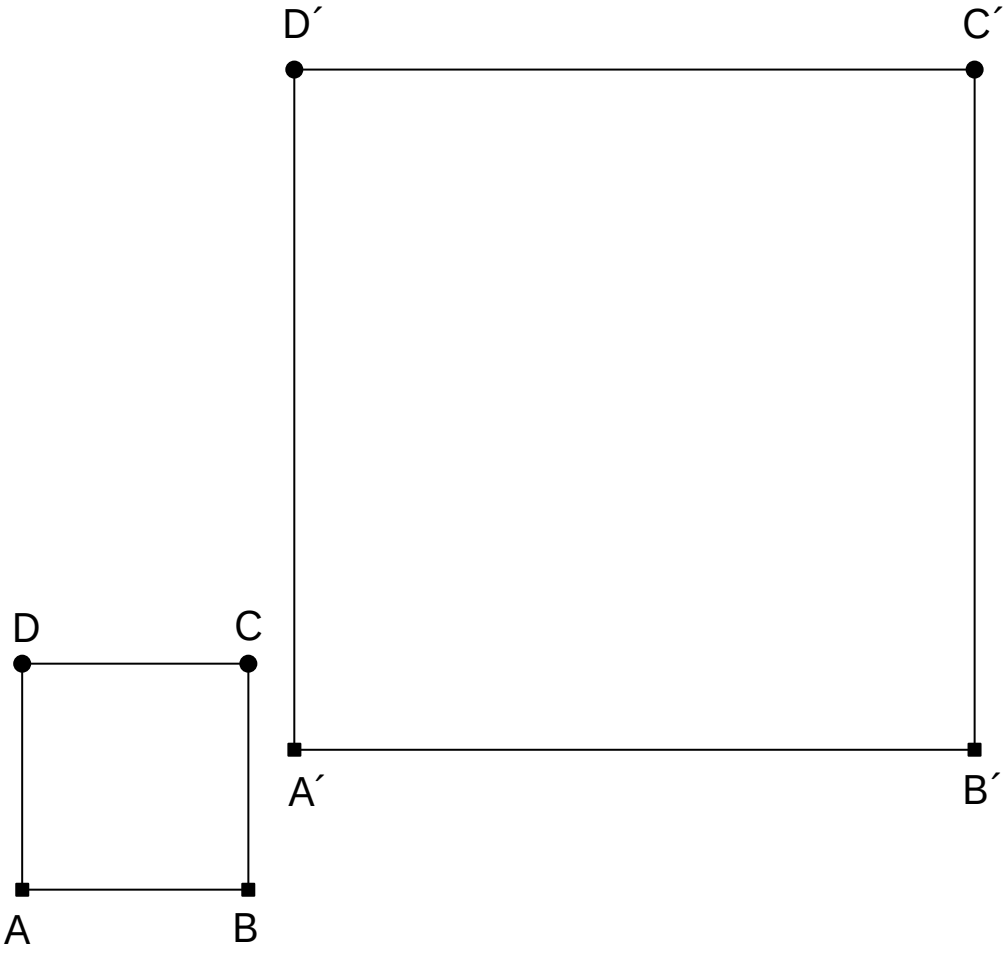
Tasks:

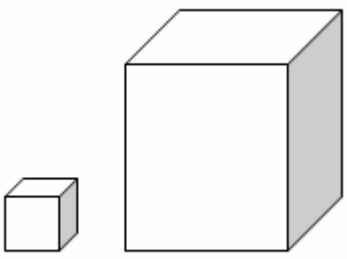
a) Determine streching center and factor (from picture).
b) Determine the ration of
   - lengths of the edges
   - areas
   - volumes of original and streched figure?

c) Describe the results of b) by using the streching factor k.


B´
A´


B
A


D´
C´
A´
B´
D
C
A
B



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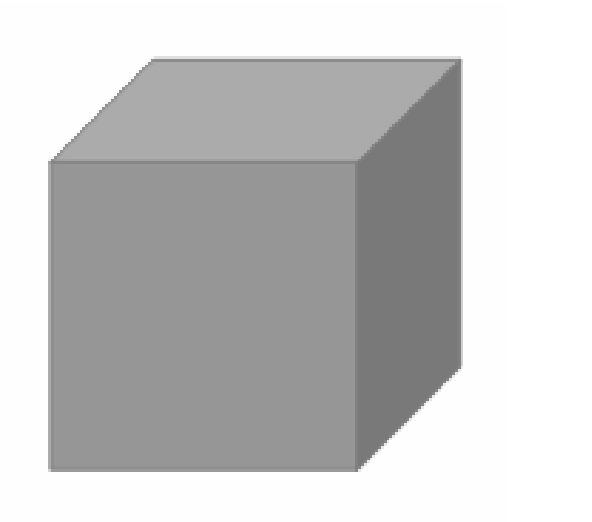
Working sheet 2:
Relation between Mass and Volumen

a) The picture shows a cube of iron with a length of the edges of 10 cm.
Calculate its mass.
(Hint: Which information do you need?).

b) This picture shows a cube of iron of a length of the edges of 30 cm.



1. Determine the ratio of the volumes of the cubes of a) and b).

2. Determine the ratio of the masses of the cubes of a) and b).

c) Describe the relation between the ratio of mass and volume of original and stretched figure in general. Use the stretching factor k.

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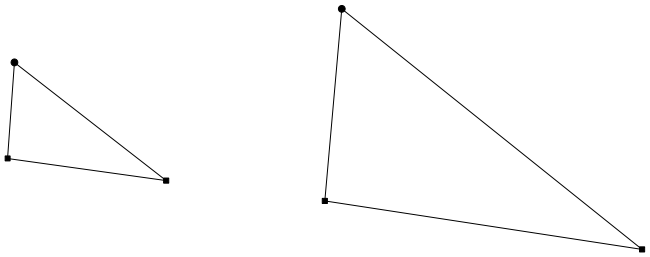
Working sheet 3: Similarity

Figures are called **similar**, if they can be mapped into each other by axial dilation

1. Show, that these two triangles are similar.



2. Which attributes do similar figures have?
   Hint: Compare side lengths, angular measure, area, etc. Say something about equalities and ratios. Check your assumptions with many different examples (For example: You can use a dynamic geometry system at the computer to create many similar figures from an initial figure).

3. Give reasons for your statements from part 2 of the exercise (for example using the creation of the similar figure with the stretching coefficient k). (In case of need use the back side).

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Summary: Similarity

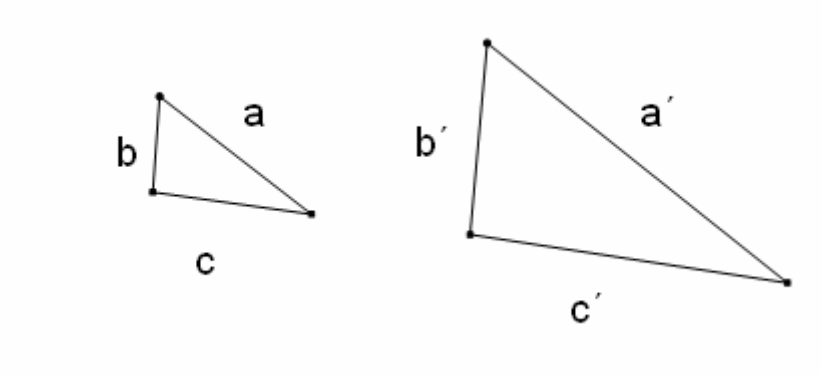
(Edge-)length of similar figures

Imagine a figure with an edge of the length a.
Strech the figure by stretching factor **k**.

Result: a similar figure.
The similar figure differs in lengths of the edges.
How long is the corresponding edge to a? _____
(Describe by using k)

Follows:
Similar figures equals in the ratio of the length of the edges.

Example:



$$\frac{a´}{a} = \frac{b´}{b} = \frac{´c}{c} = k$$

(k = streching factorr)

Or:

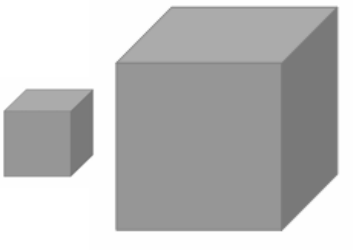
$\frac{a}{b} = \frac{a´}{b´}$ etc.

Area resp. Surface area of similar figures:

Given a figure with area A.
The figure is stretched with the stretching coefficient k.

A similar figure emerges.
This similar figure has got the area _____.
(Express the result using k)

Volume of similar figures:



Given a figure with volume V and mass m.
It will be stretched centrically with stretching coeffiecent k.

A similar figure emerges.
This similar figure has got the volume _____ and the mass _____.
(Express the result using k)

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Working sheet 4: Are there similarities about animals?

Execute each exercise for each animal couple:

a) Check if the animals are similar in the mathematical sense. Choose specific lengths, for example the shoulder height, back length or head diameter.

b) Check the similarity in a second way:
- Measure the shoulder height (respectively for birds the body length to the tail) for the animals and calculate the associated stretching coefficient.
- Inform yourself about the weight (mass) of the little animals and calculate the mass of the big animal using your stretching coefficient.
- Inform youself about the mass of the big animal. What do you say about the similarity between the two animals?

House cat and wildcat

Sources: www.pixelio.de, ID 39073, 194658)

a)

b)

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Acuate and broad mouth rhinoceros

Measuring scale of both animals: ca. 1: 50

Broad mouth rhinoceros (Source: www.pixelio.de, ID194494)

Acuate mouth rhinoceros (Source: www.pixelio.de, ID329148)

Budgie and parrot

Measuring scale of both animals: ca. 1:4

Budgie and parrot (Sources: www.pixelio.de, ID300459 und ID 254206)

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Working sheet 5: Allometries

For animals, there are obviously no exact similarities.
Instead, biologists talk about allometries. That describes different proportions for similar looking animals (For example for organs or members).

There are biological explanations for allometries.

Pup and adult animal

www.pixelio.de, ID 235810

Exercise:
For pup and adult animal we observe allometries. The animals seem similar, but differ in important body parts.

a) Justify from a mathematical point of view, to what extent the animals are not similar. So, confirm the allometry for the young and the old elephant. Hint: Measure different body parts.

b) Determine the stretching factor k for the shoulder heights and the stretching factor m for the head lengths. Compare. Also compare the ratio of head to body length.

c) Originating from this example, justify from a biological point of view the advantages of “non-similarity”. (Use the back side)

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Continuation: Working sheet 5

Big and small animal

www.pixelio.de, ID276762 und 271579

Exercise:
Elephant and ant are known as animals, which can carry heavy weights relatively to their body mass.

a) An african bull elephant has got a body weight of 6500 kg on average and is able to carry on his back and trunk loads of up to 1000 kg, using its tusks. (Alone with his trunk he can carry 200 kg over longer distances). An ant has got a weight of 8 mg and can carry loads of more than 80 mg.
Who can carry more relatively to its body weight?

b) Compare ant and elephant with a human being (Use the result from a)). Which loads could a human carry, if he possesses relatively the same force?

c) Confirm that elephant and ant are not similar from a mathematical point of view. Do this by measuring the body length and leg thickness. Calculate the ratio and confirm in this way the allometry.

d) Justiy from this example the advantage or relevance of the non-similarity from a biological point of view.

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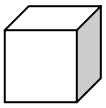
Working sheet 6: Are there bodies with same volumes but different surface areas?

Exercise 1:
What do you think about the question in the headline of this working sheet?
Discuss this in a group before solving further exercises.

Execise 2:
Imagine a cube with a border length of 1m.
This cube has got the proportions of the border lengths of 1:1:1.
(Length : Width : Height), its volume is 1 m³.



If you change the proportions of the border lengths to 1:1:2, the look of the cube changes.
If the volume has to stay constant, it becomes slim: the cube becomes a slim cuboid.
Realize that. Discuss in the group before soving further exercises.

a) Which border lengths does the cuboid have, if its volume is 1 m³ and a ratio of the border lengths of 1:1:2.

b) Sketch the cuboid true to scale.

c) Fill in the table

Proportion	Volume in m³	Surface in m²	Ratio of surface to volume
1:1:1	$1^2 \cdot 1$	6	6/m
1:1:2	$0{,}79^2 \cdot 1{,}59$	6,3	6,3/m
1:1:4			7,1/m
1:1:8			
1:1:16		10,4	
1:1:32			12,9/m

d) Draw some or all of the cuboids of the table true to scale. If you have got a mathematical software, this is done quickly.

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Working sheet 7:
Locomotion behaviour and body shapes of insects

For many insects, a specific volume is suitable for life. Depending on the way of life, the body shapes differ. A dragon fly is very slim, but a bug is more round and armoured.

www.pixelio.de, ID296549 und ID314668, Ratio ca. 1:1

Exercises

a) Estimate the volume and surface of dragon fly and bug by measuring in the picture.

Dragon fly: V = O =

Bug: V = O =

Compare.

b) Compare the locomotion behaviour of dragon fly and bug. Describe it using specific keywords.

Dragon fly:

Bug:

c) Justify the differences from a biological point of view between the characteristics from a) and b). (Use the backside).

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Working sheet 8:
Oxygen supply and body size of insects and endotherms

a) Describe the oxygen supply for insects.

b) Describe the oxygen supply for endotherms.

c) Justify from the oxygen supply, why insects are small and endotherms can become very large. In particular, comprise the ratio of surface to volume.

Hint: The heaviest animal is the goliath bug and can online have a maximum length of 12 cm.

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Working sheet 9:
Heat exchange and body size for endotherms

Shrew and a type of hummingbird are the smallest endotherms.


www.pixelio.de, ID195074 und ID279544

a) Describe the scoring effect of shrew and hummingbirg (frequency of food intake and sort of food, also in comparison to lion of human).

b) Justify the food intake. Investigate the ratio of volume to surface. Comprise the type of heat supply and heat exchange for endotherms.

c) Why do endotherms must have a minium size?