The **ScienceMath** project: **Proportions**
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# Background

## General didactical background

Starting point is an interdisciplinary approach with science, here biology. Students shall experience Mathematics reasonable, significant and interesting by extra-mathematical references; learning in contexts shall contribute to an intuitive mathematic understanding. By means of scientific contexts and methods the often watched gap between formal maths and authentic experience shall be closed on the one hand and versatility of mathematic terms shall be experienced on the other hand.

Scientific contents offer the possibility for realistic teaching. Concrete physical or biological correlations may initiate mathematical activities and lead to authentic experiences. Mathematic themes and methods are apprehended in reasonable contexts; reality of pupils may be extended by mathematical understanding. Various realistic references lead to different models and may so contribute to distinction of conceptual attributes and of different models. The variety of scientific phenomena allows open tasks and so self-dependent development of mathematics. The coherences of meanings and the differing attributes may be detected within various realistic references.

## Biological and mathematical background

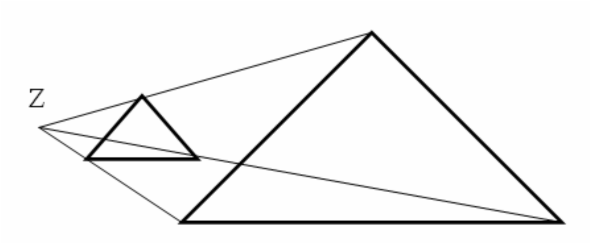
Mass resp. Volume are characteristic quantities, which allow conclusions on animal´s life and behaviour. Essential is often the ratio of volume (mass) and surface. This will be investigated in the following.

First we have to discuss the mathematical basis. They are important for the biological understanding.

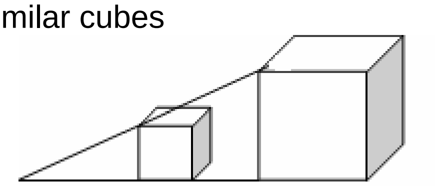
1. The concept of similarity:

Two bodies are similar, when they can be mapped into each other by a centric dilation.

Example: similar triangles




Example: Similar cubes



Similar figures equals in corresponding agles.

With strechfactor k:

- Ratio of lengths: $l_2 = k \cdot l_1$ bzw. $\frac{l_2}{l_1} = k$
- Ratio of surfaces/areas: $O_2 = k^2 \cdot O_1$ bzw. $\frac{O_2}{O_1} = k^2$
- Ratio of volumes: $V_2 = k^3 \cdot V_1$ bzw. $\frac{V_2}{V_1} = k^3$.

2. It is important to know, that figures with same volume can differ in their surface areas. Mathematical knowledge helps to understand:

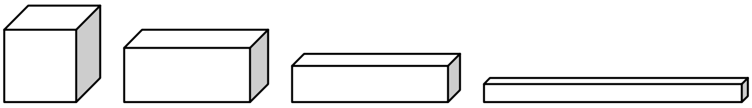
Let´s imagine a cube with an edge length of 1m, i.e. a volume of 1 m³ and a surface area of 6 m². The proportion of its edge length is 1:1:1.

Let´s change this proportion to 1:1:2. This leads to a „slimming down" of the cube to a cubold (when this cubold should have the same volume of 1 m³). Its length of the edges a resp. 2a can be calculated as $a^2 \cdot 2a = 1 \rightarrow a = \sqrt[3]{0{,}5} = 0{,}79$. Resulting surface area is about 6,3 m².

The following table shows that different ratios of length lead to bigger surface areas.
Hint: Volume and mass are proportional, at constant density.

| Proportion | Volumen/mass in m³ | Surface area in m² | Ratio between surface area and volume |
|---|---|---|---|
| 1:1:1 | $1^2 \cdot 1$ | 6 | 6/m |
| 1:1:2 | $0{,}79^2 \cdot 1{,}59$ | 6,3 | 6,3/m |
| 1:1:4 | $0{,}63^2 \cdot 2{,}52$ | 7,1 | 7,1/m |
| 1:1:8 | $0{,}5^2 \cdot 4$ | 8,5 | 8,5/m |
| 1:1:16 | $0{,}4^2 \cdot 6{,}35$ | 10,4 | 10,4/m |
| 1:1:32 | $0{,}31^2 \cdot 10{,}08$ | 12,9 | 12,9/m |

Resulting changes of the shape:



The biological consequences from the ratio of volume to surface are shown in specific shapes of the body and behaviours of the animals.

**a) There are insects with different lankiness but identical mass.**
For many insects, a specific volume is adequate. But depending on the environment their shapes of body differ. A dragon fly is, for example, very slim to fly fast. However, a bug is more round and armoured, because it is living on the ground.

**b) Insects are small.**
Trachea maintain the oxygen supply for insects. These are pipe systems in the chitinous exoskeleton. The exchange occurs at the body surface of the insects. The heavier the insect the more oxygen it needs. From ca. 15 cm length, the ratio of volume to surface is becoming so disadvantageous that the oxygen supply does suffice. The heaviest insect is the goliath beetle that has got a maximum length of 12 cm.

**c) Animals can become very large if the oxygen supply is carried out by the blood.**
The amount of blood increases with the volume of the animal. The more blood flows, the more oxygen can be transported. So, there is no disaccord as with the insects.

**d) The endotherm must have a minimum size.**
Small animals have got a low body mass and thus a small amount of blood that provides heat for the body. The surface is constantly radiating heat. The smallest mammal, for which the ratio of volume to surface does suffice, ist the shrew. The smallest mammal for that the ratio from volume to surface does suffice is the shrew. The smallest bird is a type of hummingbird (2,5g).

**e) Small endotherms have to eat high-energy food all the time.**
If the ratio of mass to surface is so unfavourably like fort he hummingbird ort he shrew, the animal has to absorb energy constantly to survive. In fact, these little animals eat all along namely preferably energy-rich nectar. On the other hand, animals with a better ratio, like for example the lion or humans, may sustain hours without nourishment.

**f) small and big animals differ in their relativ body strength.**
Small and big animals appear quite similar. But they vary in several attributes relatively. For instance in the ability to carry huge masses according to their body mass.

Example:
A shrew has got a length of 4,3 cm, an elephant 3,5 m.
This results in a stretching factor of 814.

When comparing the masses of an elephant (4000 kg) and a shrew (2,5 g), one can see that they do not have the same proportions. In particular, this can be seen from the disproportional enlargement of the cross section of the muscles, for example of the legs, to carry ist own body weight. Thus, absolutely, an elephant can carry more weight than a shrew, but relatively much less.

**f) Allometries in animality**
In the animality there are obviously no exact similarities between animals. Instead, biologists talk about allometries. This describes different proportions of organs and members of animals with similar appearance. By comparing the masses, measurements with living

animals or using photographies, one can calculate the allometries of similar animals like cat and tiger or pup and parent animal.

Example: $\frac{m_{Tiger}}{m_{Hauskatze}} = \frac{100kg}{5kg} = 20$ Stretching factor: $k = \sqrt[3]{20} = 2,7$

The shoulder heights behave like 1:2,7, but the surfaces like 1:7,3.

*Source: www.pixelio.de, ID 39073, 194658)*

Allometries often have biological reasons, for example the different importnce of inner organs, or the scheme of childlike characteristics: That pups have bigger heads to seem cuter to trigger a sense of protection in the parent animals.

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