The **ScienceMath**-project: **Slipper Animalcules**
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Teaching Material

Worksheets and tasks (see next pages)

This project has been funded with support from the European Commission. This publication reflects the views only of the author(s), and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Slipper animalcules (*paramecia*) are single-celled organisms that live in water and have appropriate living conditions in many locations on the planet. They are characterised by their exterior being covered by small cilia that constantly move by beating in a specific direction. The primary function of the cilia is to propel the paramecium forward in the water. Slipper animalcules feed on the bacteria from decaying organic matter. Slipper animalcules play an important role in the ecosystem as food for smaller animals such as larvae.

Slipper animalcules can reproduce themselves both through cell division and gendered conjugation much like the transmission of genetic material found in more complex animals.

Assignment 1: Fill out Table 1 on the last sheet. (It is ok just to stop filling out when you get the overall impression). Afterwards you must sketch the graphs of

a) N in a (N, t) system of coordinates

b) $N'(t)$ in a $(N'(t), t)$ system of coordinates

c) $\frac{N'(t)}{N(t)}$ in a $\left(\frac{N'(t)}{N(t)}, N(t)\right)$ system of coordinates

Which relations do the graphs show, and which information does this provide about the population of Slipper animalcules in the experiment? How can you find out whether what we witness is an example of logistic growth?

Assignment 2: How do you imagine a graph which describes the growth speed of the growth speed $N'(t)$ looks like? And what can this information be used for? Feel free to use an example – such as $f(x) = x^3$ - and ask yourself: “What is the differential quotient of $f'(x)$ - i.e. the differential quotient of the differential quotient of $f(x)$?”

Assignment 3: When you are modeling the population of e.g. Slipper animalcules, and you have identified that the type of growth is logistic growth, you can try to make this fundamental model to fit your data:

$$N(t) = \frac{K}{1 + \left(\frac{K - N_0}{N_0}\right) e^{-rt}}$$

This equation is called *the logistic equation*. The parameters are defined as follows:

$N(t)$	The population at time t
K	The maximal population capacity which the surrounding environment can sustain
N_0	The initial population – i.e. $N(0)$
r	The growth rate of the population

By looking at Table 1 you can find good suggestions for the values of K and N_0 – try to venture a guess of the value of r and see if the graph of your model fits the data.

Why does the logistic equation look as it does?

Let's begin by asking "what determines how a population grows?". First there is the relation between how many individuals (Slipper animalcules) that dies per unit of time and the number of Slipper animalcules being born per unit of time. It is this relation which is often called the population rate r and this parameter describe the increase in the number of Slipper animalcules in the existing population per unit of time. A bit simplified you could say that the *changes in the population at a given time t* is given by $rN(t)$. But then the population would grow with a constant speed. And that does not conform to the data of reality, because a population typically grows slowly at the beginning (because the low number of individuals) and then more and more quickly until it again begins to slow down (because the environment only affords a maximum capacity – what we have called *K*). One way to describe the growth, then, is to say that the population at any given time *t* is given by $rN(t)\left(1-\frac{N(t)}{K}\right)$. You can try for yourself to think about what effect the expression $\left(1-\frac{N(t)}{K}\right)$ has (investigate what happens both when $N(t)$ is very small and when it is almost as big as *K*). But since $rN(t)\left(1-\frac{N(t)}{K}\right)$ is an expression of the *change* in population it follows that $rN(t)\left(1-\frac{N(t)}{K}\right)=\frac{dN(t)}{dt}$. And the logistic equation is the antiderivative of $\frac{dN(t)}{dt}$ – i.e. the logistic equation is an expression of $N(t)$.

Assignment 4: (a) Describe in words what you understand by the word "function". Mention how functions often are used in connection to modeling and describe which information you can get from

i. $f(x)$ in a $(f(x),x)$ system of coordinates

ii. $f'(x)$ in a $(f'(x),x)$ system of coordinates

iii. $\frac{f'(x)}{f(x)}$ in a $\left(\frac{f'(x)}{f(x)},f(x)\right)$ system of coordinates

(b) Describe in words the different possibilities provided by the use of differential calculus and integral calculus when working with models.

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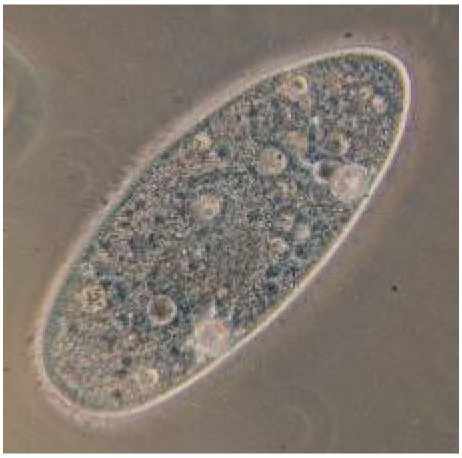
Table 1:

Hours **T**	Populationsize **N**	Estimated values of the growth speed
0	2	
3	3	
6	4	
9	5	
12	6	
15	8	
18	11	
21	15	
24	19	
27	25	
30	33	
33	43	
36	55	
39	70	
42	88	
45	108	
48	132	
51	158	
54	184	
57	211	
60	237	
63	261	
66	283	
69	301	
72	317	
75	330	
78	340	
81	348	
84	355	
87	359	
90	363	
93	366	
96	368	
99	370	
102	371	
105	372	
108	373	
111	373	
114	374	
117	374	
120	374	
123	374	
126	375	
129	375	
132	375	



From Wikipedia