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Education and Culture
Lifelong learning programme
COMENIUS

Background

General Didactic Background

Background of this teaching module is that students shall experience Mathematics reasonable, significant and interesting by extra-mathematical references; learning in contexts shall contribute to an intuitive mathematic understanding. By means of scientific contexts and methods the often watched gap between formal maths and authentic experience shall be closed on the one hand and versatility of mathematic concepts shall be experienced on the other hand.

Basic idea is that the students shall learn the concept of parallelism in authentic situations and references through action or action-oriented learning. The situations in the experiments lead to the concept of parallelism through common aspects and differences in a surprising way. The students get aware of the central characteristic of equal distances.

Mathematical Background

In the Eucledian geometry two straight lines are called parallel if they lie in one plane and do not have one point in common. The parallel axiom states that there exists to a straight line g and a point P exactly one straight line h to g through P which is parallel to g.

Definition:
Two straight lines g and h are called parallel, if: The distance of all points of g to h is constant.
The distance of a point P of a straight line g or a plane E is the length of the line segment PF, if F is the foot of the perpendicular that is dropped from P to g respectively to E (figure 1).
A corresponding definition applies also for planes.

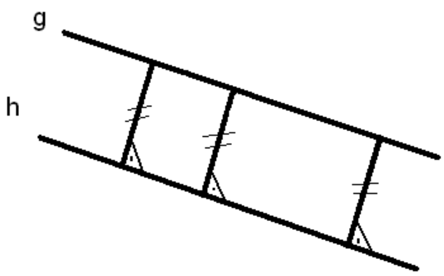

figure 1: Proof of parallelism by equality of the perpendicular distances of various points of g; equality of distance of the straight lines g and h.

However applicable knowledge about parallel terms will not only be obtained through definition but also by bringing parallelism into combination with other mathematic objects. These are the theorems of angles like the theorems about alternate angles, the corresponding angles as well as the theorem of intersecting lines resp. their inversions.

An exciting connection arises regarding the subject circle. A circle is the set of all points that have the same distance r from a given point M. For this reason the length of all circle chords through M is always equal, i.e. the distance of the two intercepting points of each diameter straight with the circle periphery. From this it follows that two straight lines which are circle tangents for two circles with the same diameter are parallel. Contrariwise two circles between two parallels have to have the same diameter. In further consideration objects between two parallels have to have the "same thickness" (figure 2).

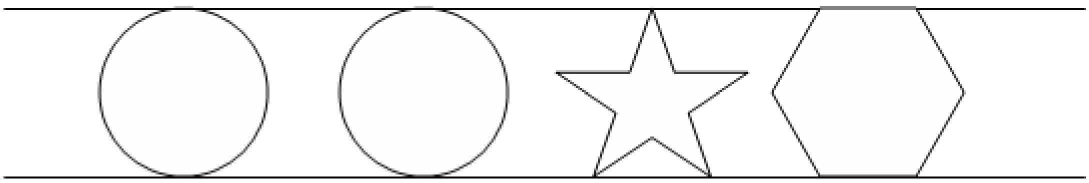

figure 2: Objects of "same thickness" between two parallels

Particularly interesting becomes the context if considering things dynamically. Beside a circle further objects do exist where parallel straights stay parallel under their movement? How do these objects have to look like and what features do they have to have?
To answer that, the definition of parallelism plays an important role (see also Orbiforms/ "same-thicks" at 4).

Regarding the question application oriented: parallelism can be proved practically by the use of spirit levels. In general it is interesting to see whether a board or a wall is parallel to the earth surface or to its vertical, i.e. exactly horizontal or vertical.

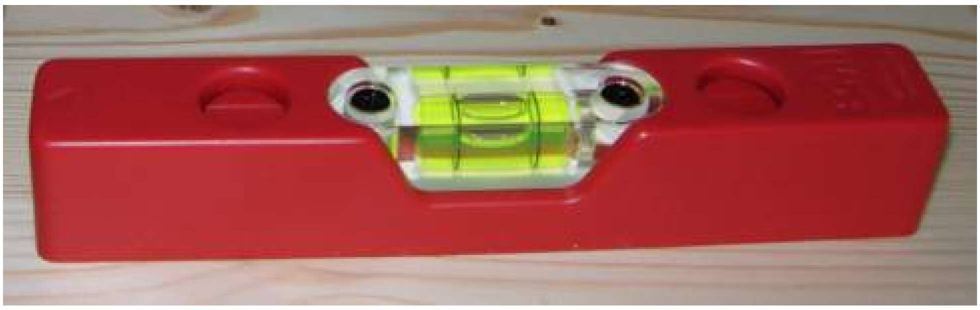

figure 3: spirit level.

The position of the board is horizontal, because the air bubble in the water is seen between the two marks. The surface of the water adjusts to the (thought) earth level. The parallelism to earth level stays also when the spirit level is moved. In case of a tipping movement the surface of the water doesn't stay parallel to the ground of the spirit level. The air bubble swims off the marks. When the ground of the spirit level is fixed parallel to the measured surface, we get information about horizontality from the position of the air bubble.

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