Education and Culture
Lifelong learning programme
COMENIUS

Teaching Material

Parabola and Car Lights

Parabola has besides its wide use and known analytical expression also much older and even more elementary accessible geometric approach. By simple geometry we can explain technical functioning of a car headlight and of a satellite dish.

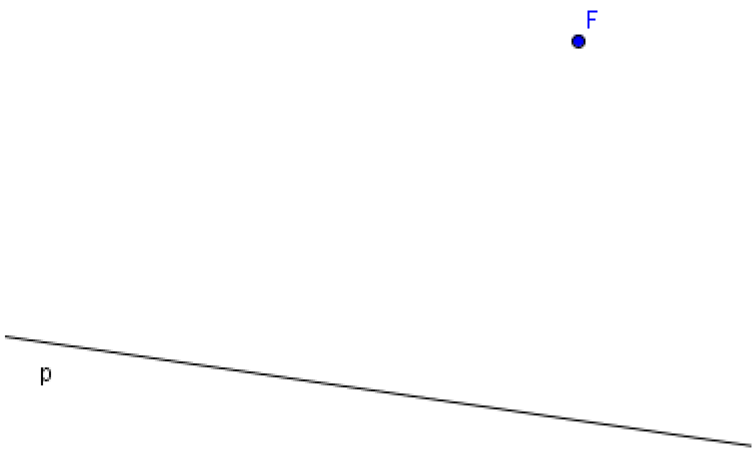
Here we present the ideas in a sequence of content, which a teacher can adopt for his/her students. If using interactive computer equipment, the theoretical geometric introduction should be done later when the need for understanding and proofs appears. Also in a more classical presentation, we advise to motivate the subject by posing the right questions according to the level of student's knowledge. The lecture could start with a discussion of car lights and its high and low beam…

A web version of the lesson with useful interactive simulations can be found at
http://uc.fmf.uni-lj.si/com/Parabola/parabola.html.

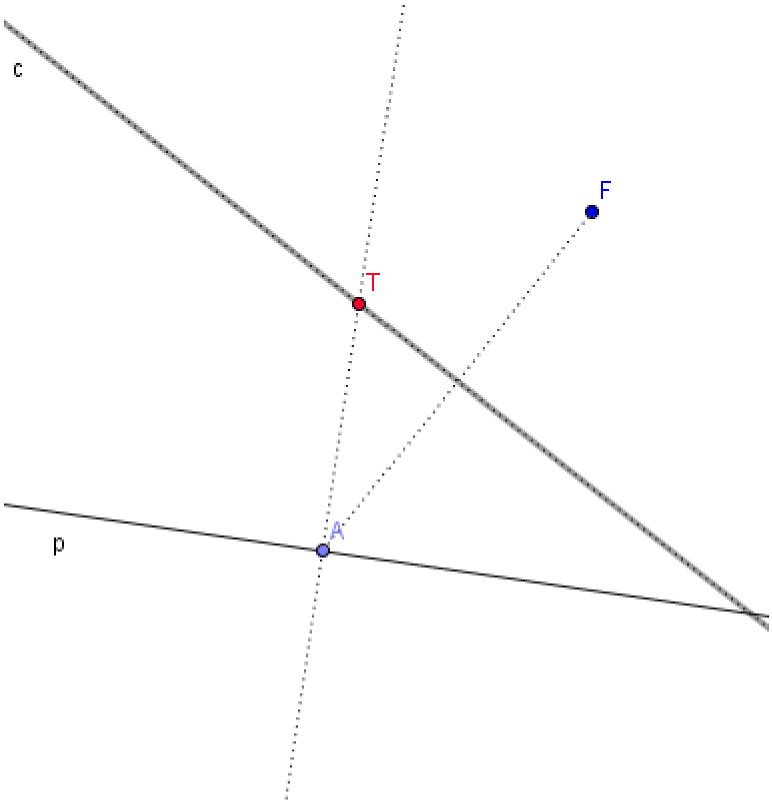
THEORETICAL GEOMETRIC INTRODUCTION

We start with ancient geometric definition of a parabola.

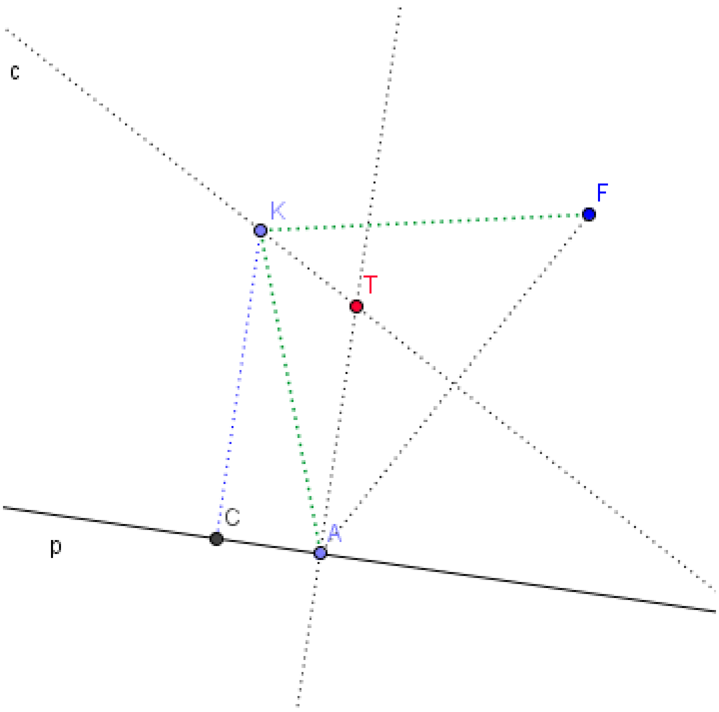
A parabola is the set of points in the plane that are equidistant from a point (the focus) and a line (the directrix).



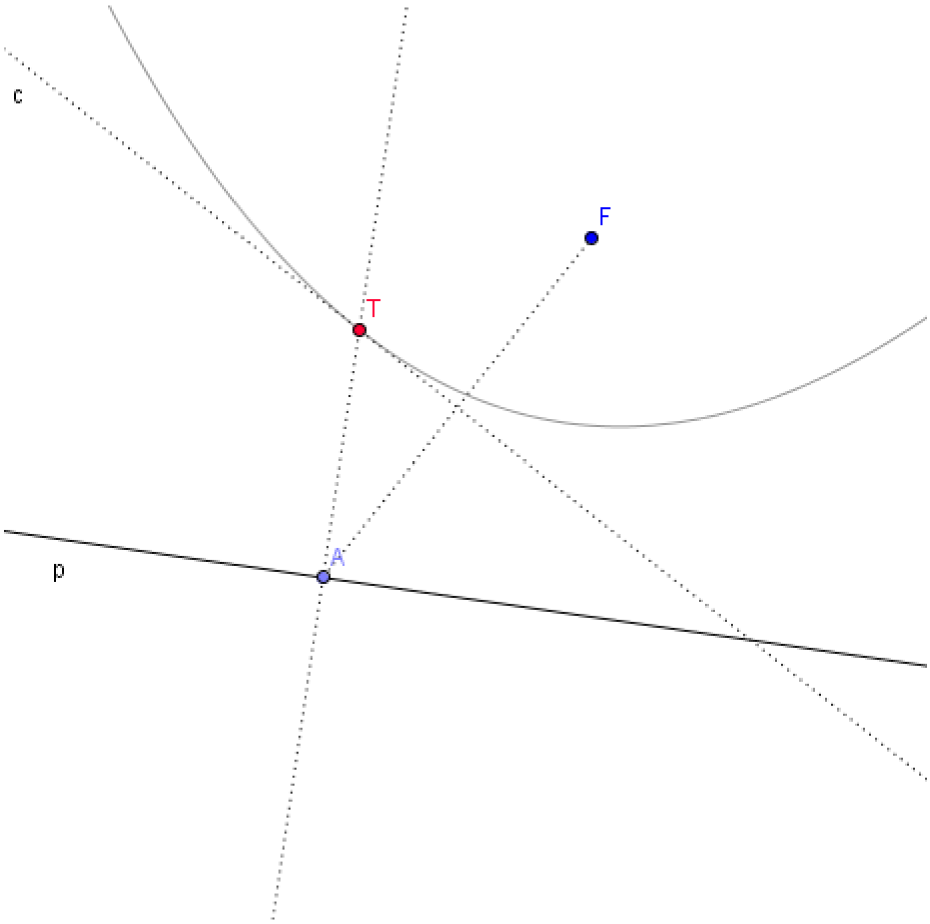
To see which points lay on the parabola, we start with a point *A* on the line and draw the segment *AF*. A perpendicular line to *p* at *A* and a perpendicular bisector *c* of the segment *AF* intersect at point *T*.



Point *T* lies on the parabola since triangle *AFT* is an isosceles triangle. Furthermore, the bisector *c* is a tangent to the parabola. Say *c* is not a tangent. Since point *T* lies on the parabola, there should be another point *K* on *c* and on the parabola.

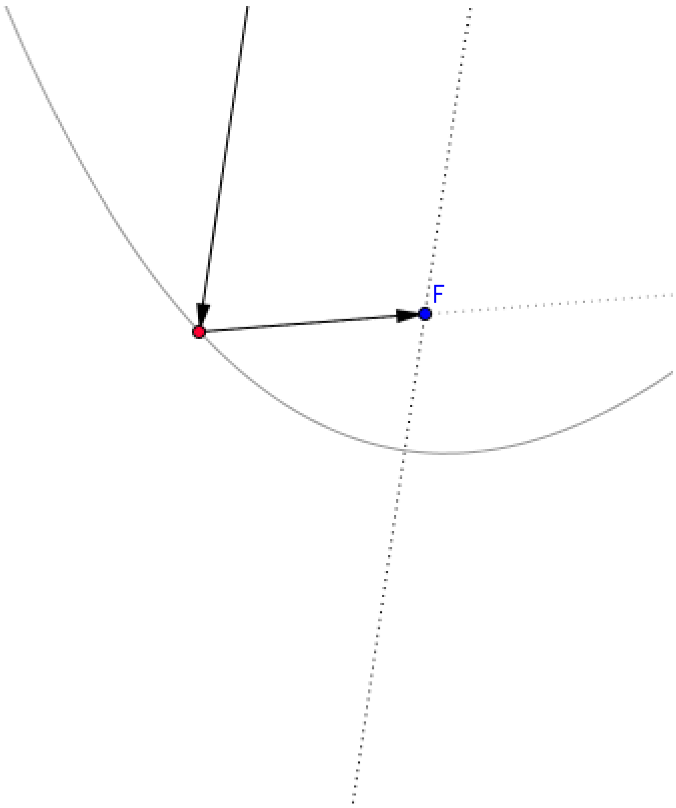


But since the triangle *AFK* is also an isosceles triangle, *KF* can not be equivalent to *KC*. Therefore, line *c* is a tangent to the parabola. As we move the point *A* up and down the line *p*, the point *T* travels along the path of the parabola.

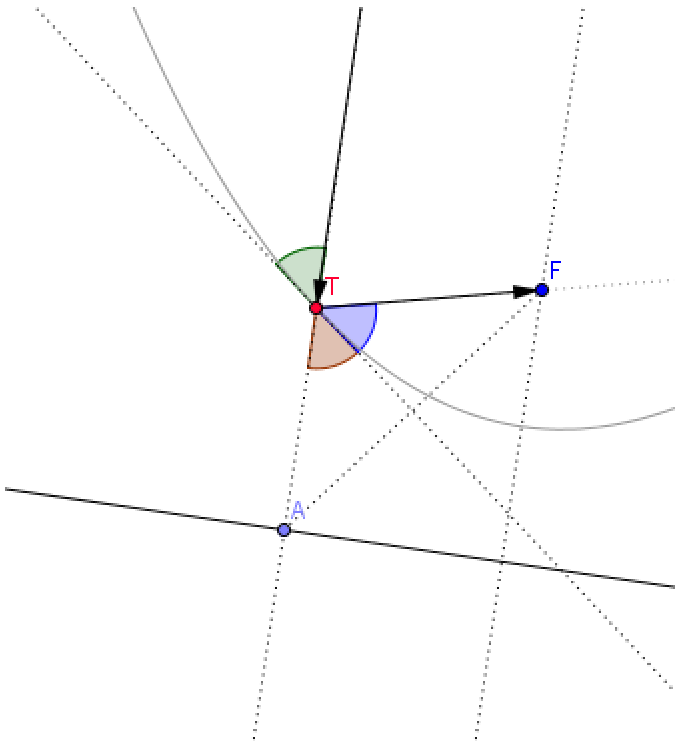


PRACTICAL IMPLEMENTATION

How a ray (or a billiard ball) falling on parabola along the axis of the parabola reflects/bounces off. This 'experimenting' can be done by guessing, sketching, exact drawings and finally by the use of dynamic geometry programs. Does it only seem so or is it true, that the ball/ray always bounces off towards the focus of the parabola?

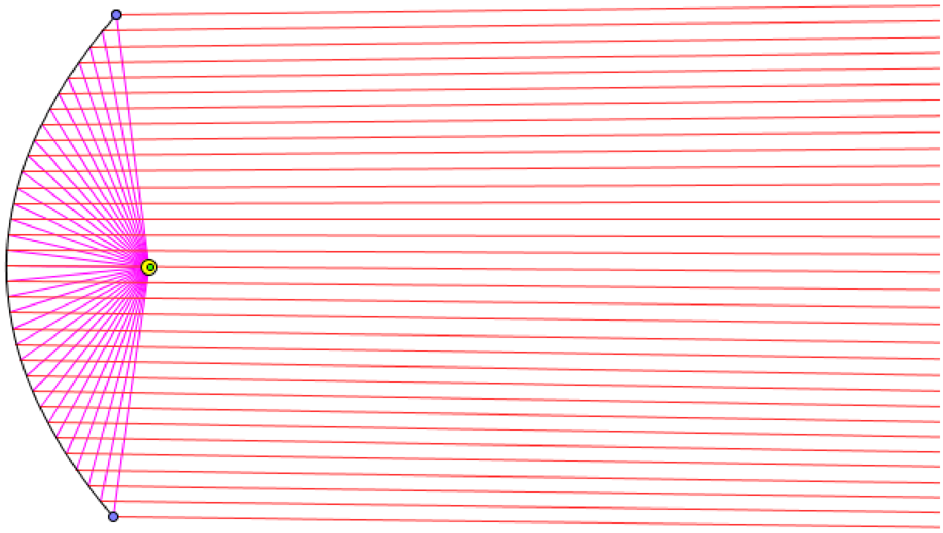


If we return back to our original drawing and recall how we got a point *T* on the parabola, it is not hard to conclude, that ‘green’ and ‘brown’ angles are equivalent as they are *vertical angles*, while ‘brown’ and ‘blue’ are equivalent because triangle *AFT* is an isosceles triangle. Therefore, it becomes obvious, that a ray parallel to the parabola axis bounces off the parabola exactly towards the focus of a parabola.

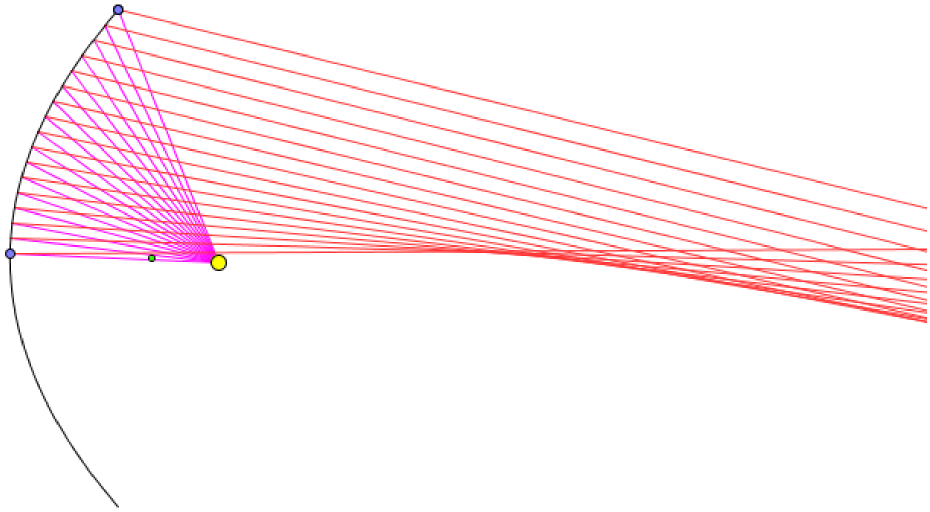


CAR HEADLIGHTS

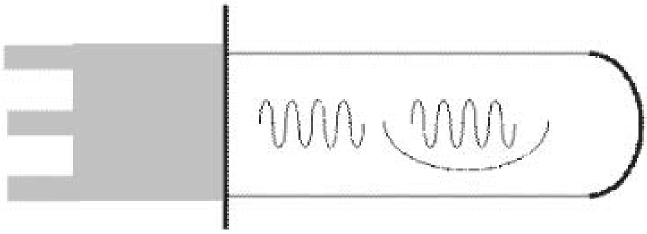
This property is used in the functioning of car headlights. Car lights have a ‘shape of parabola’. When the source of light is exactly in the focus of a parabola we get the high-beam:



When the source of light is a bit ahead of the focus of a parabola and the source of light is shaded from bellow we get the low-beam:



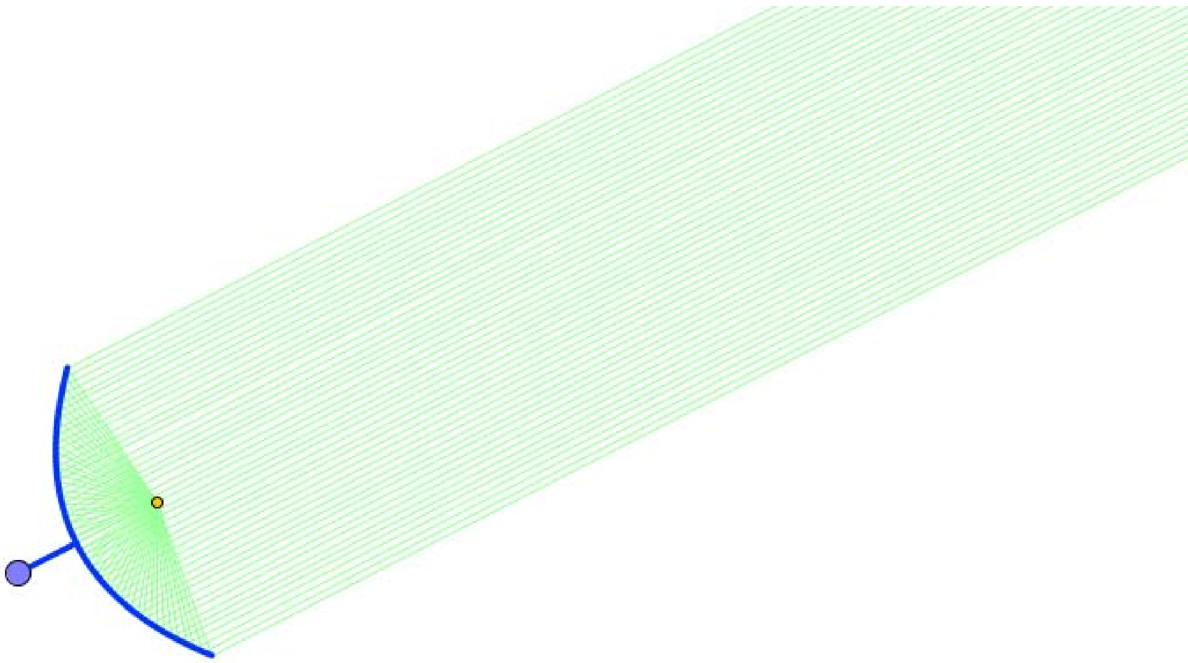
But how is this done mechanically? On this stage it is advisable to show a real car headlight bulb (can be a broken one) and analyse and compare it to what was just said. On the following picture we see a photo of a real car headlight bulb and a simplified drawing. Notice the two wires. Low-beam wire, on the right, has a ‘cap’ underneath. High-beam wire, on the left, is placed exactly to the focus of the parabola. Just a couple of millimetres distance between the ‘high-beam and low-beam wire’ in a headlight bulb and the property of a parabola make the difference between high and low beam.



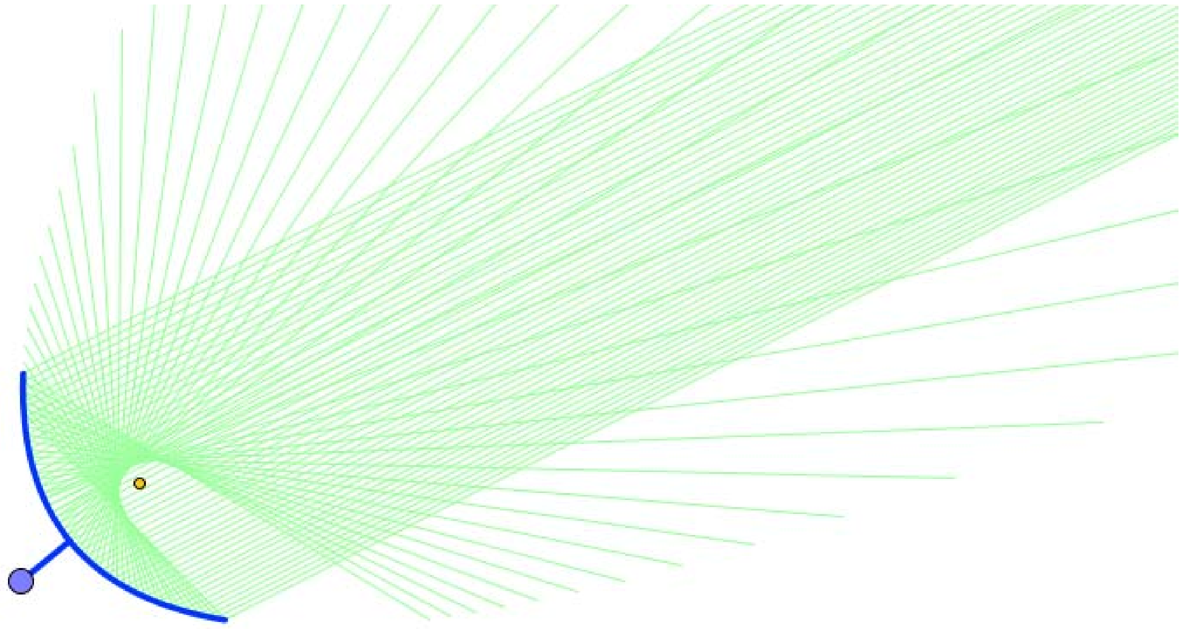
At this point a practical approach really pays off. Teachers familiarity also with the technical aspects of the idea is very important. Students understanding of the idea can be nicely tested and stimulated by different questions. For example, why does the bulb have the 'metal cap' in front? Students can be challenged to figure out at home (on the way home, observing cars, asking a mechanic, ...) whether all the bulbs are of the same shape? Namely, some headlight bulbs do not have the metal cap in front, but the according headlights already include the 'front cap', which can be easily seen in the headlights of some modern cars.

SATELLITE DISH

The very same idea is used in the satellite dish. When a satellite dish is directed towards a particular satellite, all the beams that fall on the dish are reflected to the focus of the satellite dish where a sensor 'picks up' all the strengthened signal. Why do the signals from a satellite travel to the dish in the lines parallel to the axis of the parabolic dish? Well, it is an approximation, as the satellite is so far away, the beams that fall on the dish are practically parallel.



If the satellite dish is not correctly positioned, lots of ‘rays’ are lost, because they are reflected away from the focus and the sensor. Consequently, the TV reception is poor.



Depending on the students and the skills of the teacher we can proceed into the discussion of the satellite dish shape. Namely, we know different parabolas. What happens if we take a wider or a narrower parabola? Why satellite dish shape is a rather ‘wide parabola’?

Remark. Regarding the level of students we can intertwine the geometric ideas presented with a more common analytic approach to parabola. Can we give analytic meanings and proofs for everything said? Can we compare the two approaches and advantages of each?

Interactive simulation

Some interesting Interactive computer simulations of the ideas presented can be found and studied on the web.

This project has been funded with support from the European Commission. This publication reflects the views only of the author(s), and the Commission cannot be held responsible for any use which may be made of the information contained therein.