ScienceMath-projekt: **x^-0,5**
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Teaching material

A natural phenomenon is observed and the measured points are plotted in graph. The question is: what mathematical function fits the plotted points and therefore describes this phenomenon.

We start using one of the computer applets
(http://www.hanksville.org/courseware/solarsystem/planets.html

http://galileoandeinstein.physics.virginia.edu/more_stuff/flashlets/innerplanets.htm

http://physik.uibk.ac.at/hephy/applets/physlet_resources/bu_physlets/c17_solar_sim.html)

which simulate the movement of inner solar system planets. The orbits are presented by cycles, although we know that planets path around Sun are ellipses. As the eccentricity of the ellipses is very close to 1, this approximation is justified. The central question is: what is the relationship between the speed of the planet and the radius of its orbit. We plot the measured values into speed-radius coordinate system and try to guess the appropriate function which ties the two variables.

We project the computer applet to the whiteboard and measure the radii of the orbits. My suggestion is to express all the radii in the unit of astronomical unit (distance Earth-Sun). This is the independent variable and will be plotted to the horizontal axes. Using a stop-watch we also measure the periods of the planets. Maybe we calculate all the times into a year. And finally we calculate the speed of a planet:

$$v = \frac{2\pi r}{t_0}$$

The horizontal axis is reserved for the velocity. The unit is au·2π/year which is the length of earth's orbit divided by year. We insert the four measured velocities and try to fit an appropriate curve through the points. It is most probable that students will first suggest plotting 1/x. As it is not the right one, they sometimes try with $1/x^2$. The right one is $1/x^{1/2}$. The attached Excel file can serve to plot them easily (See attached file). The function definition range is from 1/10 au.

Two possible explanation of physics background

Starting from Kepler's law

In the case we can assume that the planets orbit the Sun on circles and not on ellipses, we state the third Kepler's law as:

The ratio between the orbit of a planet and its period is the same for all planets:

$$\frac{r^3}{t_0^2} = konst.$$

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We divide both sides by r and multiple by $(2\pi)^2$. The left side of the equation is the magnitude of the velocity of a circling object squared.

$$\frac{(2\pi)^2 r^2}{t_0^2} = \frac{konst.(2\pi)^2}{r}$$

Therefore we introduce another symbol:

$$v^2 = \frac{konst.(2\pi)^2}{r}$$

Taking the square root of both sides we obtain:

$$v = 2\pi\sqrt{\frac{konst.}{r}}$$

Starting from Newton's law

Again, we assume that the orbits are perfect circles. Therefore the centripetal force is written as:

$$F_c = m_p \frac{v^2}{r}$$

In this equation m is mass of the planet, v its velocity and r radius of the orbit. We are fully aware that centripetal force is only a name for the sum of all forces acting on a circling body. The effect of that net force is permanent change of the direction of the velocity while its magnitude remains the same. There is only one force acting on the planet: that is the gravitational force of the Sun. It is Newton's discovery that there is a force acting between two bodies and this force can be written as:

$$F = \frac{Gm_s m_p}{r^2}$$

G is the universal gravity constant, m is mass of the planet and sun respectively and r distance between the centers of gravity of the two bodies.

We combine the two equations

$$m_p \frac{v^2}{r} = \frac{Gm_s m_p}{r^2}$$

Using basic algebra we rearrange the equation into: $v = \sqrt{\frac{Gm_s}{r}}$

The result explicitly shows that the speed is proportional to the distance to power -1/2.

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