The **ScienceMaths** project: **Logarithmic function**
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Education and Culture
Lifelong learning programme
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Background

General didactic background

In math lessons the teacher tells the definition of logarithms and then derives some rules of calculation with them. Students learn the definition and rules and how to solve typical math problems[1]. But after passing the subsequent test or exam, they then forget almost all knowledge of logarithms. The concept of the logarithm is not related to any other mathematical concept. It is only connected with the exponential function[2], but the exponential function is also flying around somewhere in space.

Students can write down equations and apply them to three or four routine types of textbook problems. This is not convincing evidence that the student really understand the theory.[3] Perkins claims that knowledge and skill themselves do not guarantee understanding.[4] Rote knowledge generally defies active use, and routine skills often serve poorly because students do not understand when to use them. If knowledge and skills are not understood, the student cannot make good use of it.

We can make the learning of logarithms more effective if we combine the classical way of teaching with new approaches where students are shown the applicability of logarithms also in everyday life, outside the world of mathematics. They can build up a network of many concepts, properties, applications, etc. Collins and Loftus[5] speak of “the spreading activation model”. This model assumes that stored knowledge is best thought of as a network of multiple interconnected and related data where the processing of an item leads to the activation of other related items. The activation process tends to spread in all directions in order to activate the related information.

Therefore, our aim is to build a network of many interconnected concepts that includes the logarithm. We must teach for understanding in order to realize in long-term payoffs of education.[6]

At first students are not so enthusiastic about such a method because it is difficult for them to transfer concepts, ideas and procedures learned in mathematics to a new and unanticipated situation in science.[7] They have to use knowledge from other subjects, such as physics, chemistry, biology, or psychology. And they must become active. They do a lot of experiments where they have to use math’s concept of the logarithm. But later on, active participation brings a feeling of success to them, along with motivation. And learning in context contributes to an intuitive mathematic understanding.[8]

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[1] E.g.: Draw the graphs of a logarithmic function, solve logarithmic equations, etc.
[2] It is defined as the inverse function of the exponential function.
[3] Teaching for understanding (Perkins, 1993)
[4] Teaching for understanding (Perkins, 1993)
[5] Spreading Activation Model of Semantic Memory (Collins & Loftus, 1975).
[6] Teaching for understanding (Perkins, 1993)
[7] Functions: a modeling tool in mathematics and science (Michelsen, 2006)
8 Research considerations for interdisciplinary work on mathematics and its connections to the arts and sciences (Beckmann, A. & Michelsen, C. & Sriraman, B., 2005).

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Mathematical background

Logarithm function is first introduced as an inverse function of exponential function:

$\log_a x = y \Leftrightarrow a^y = x$, where *a* can be any positive real number.

Logarithm is defined only for positive number *x* because a^y which is equal *x* is always positive, too. To be specific, the logarithm of a number *x* to a base *a* is just the exponent you put onto a to make the result equal x.

In general, if $x = a^y$ then we say that "y is the logarithm of x to the base a".

We said that any positive number is suitable as the base of logarithms, but two bases are used more than any others:

base 10: $\log_{10} x = \log x$ we call it common logarithm

base e: $\log_e x = \ln x$ we call it natural logarithm[9]

From the definition we can derive that:

$\log_a 1 = 0$ because $a^0 = 1$

$\log_a a = 1$ because $a^1 = a$

$\log_a a^x = x$ because $a^x = a^x$

And we also derive the rules of logarithmic computation:

$\log_a (xy) = \log_a x + \log_a y$ [10]

$\log_a (x/y) = \log_a x - \log_a y$

$\log_a x^m = m \log_a x$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Logarithmic function is defined as $f(x) = \log_a x$ for $x \in R^+$.

Graph of logarithm and exponential function are symmetrical[11] to y = x:

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[9] The number *e* is called Euler's number after the Swiss mathematician Leonhard Euler, or Napier's constant in honor of the Scottish mathematician John Napier who introduced logarithms. Since *e* is transcendental, and therefore irrational, its value cannot be given exactly as a finite or eventually repeating decimal. The numerical value of *e* truncated to 20 decimal places is: 2.71828 18284 59045 23536... Otherwise it is defined as $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$ or as the sum of unlimited series:

$$e = 1 + \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \ldots$$

[10] Multiplication can be performed by addition – this is the biggest advantage of logs. It is similar idea as in the formula: $\sin\alpha \cdot \sin\beta = \frac{1}{2}(\sin(\alpha - \beta) - \cos(\alpha + \beta))$.

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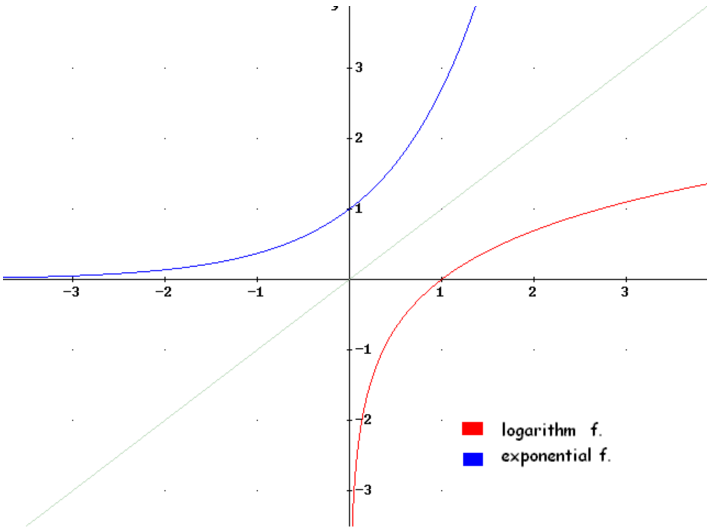

Graph 1: Graph of logarithm and exponential function

Why and where do we use logarithms today?

a) To model many natural processes, particularly in living systems. We perceive the loudness of sound as the logarithm of the actual sound intensity, and dB (decibels) is a logarithmic scale. We also perceive the brightness of light as the logarithm of the actual light energy, and star magnitudes are measured on a logarithmic scale.
b) To measure the pH or acidity of a chemical solution.
c) To measure earthquake intensity on the Richter scale.
d) To present on the same graph very small and very big numbers. We use a logarithmic scale when there is a wide range of values.
e) Psychologists draw a curve of forgetting[12] which is similar to the graph of a logarithmic function. And they also claim that our lives do not run in a linear fashion and that our perception of time is logarithmic.[13]
f) To analyze exponential processes.[14] Applications include the cooling of a dead body, the decay of radioactive isotopes, and the growth of bacteria. The spread of an epidemic in a population often follows a modified logarithmic curve called a “logistic”.
g) To find the number of payments on a loan
h) To solve some forms of area problems in calculus.[15]

Some of them are explained in teaching material.

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[11] Two inverse functions has always symmetrical graphs to the line y = x.
[12] The curve of forgetting describes how we retain or forget information that we take in.
[13] For example: the years from ages 10 to 20 seem to pass in the same amount of time as the years from 20 to 40, or those from 40 to 80.
[14] Because the log function is the inverse of the exponential function, we often analyze an exponential curve by means of logarithms.

[15] The area under the curve $1/x$, between x=1 and x=A, equals *ln* A.

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The idea of teaching implementation

In fact, this approach as well demands good preparation of math teachers, who have to learn some topics from other subjects, too and to cooperate with other teachers. Although a growing number of subjects include ingredients form mathematics, it is still difficult for both teachers of mathematics and of other subjects to see the use of mathematics in other subjects – partly due to the use of concepts and language.[16] The best way is to have both teachers present (e.g. math and physics teachers) at the same time in the same classroom and combine their lectures.

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[16] Functions: a modeling tool in mathematics and science (Michelsen, 2006)