

The **ScienceMath**-project: **Horizontal Launch and Parabola**
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Teaching Material

Parabola between Mathematics and Physics - The Case of the Horizontal Launch

Proposal for a possible course

See next pages

Introduction

The Module consists of two parts.

The part *FROM PHYSICS* is executed by the physics teacher, the part *FROM MATHEMATICS* is part of the math teacher.

FROM PHYSICS ...

Introductory experiment

Kinematics is the branch of classical mechanics concerned with describing the motions of objects without considering the factors that cause or affect the motion. After mastering the linear motion we focus on the horizontal launch. This plane movement is studied quite in detail. The first experiment is well-known two coins experiment (Figure 1).

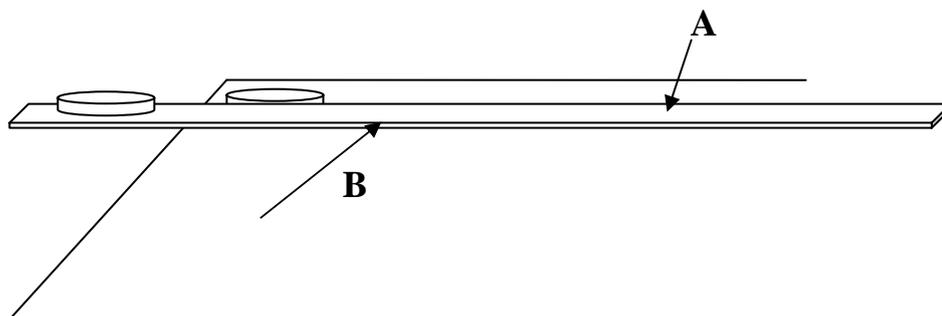


Fig. 1.

The set-up is placed on a high object (for example a 2 meter high wardrobe). One finger is pressing at point A while the other will rapidly push (snap) the ruler at point B. Sudden movement of the ruler will cause the left coin to fall down as a freely falling object while the right one will be launched in horizontal direction.

Preparation

Before carrying out this experiment, students are asked to discuss and guess the experiment's possible outcome. Very few of them predict that the coins will hit the floor at the same time.

Carrying out the experiment

After a few repetitions of the experiment, they agree, that one can hear only one "clap", therefore that the two coins are falling downwards with equal speed.

Conclusion

We conclude: movement of a horizontally thrown object can be split into two independent movements: one is a vertical movement which is equal to a freely falling body movement (the two coins began their path at the same level and time and they simultaneously hit the floor). But this simple experiment can not tell us what is happening regarding the horizontal part of the coin's movement.

The vertical movement can be calculated by the same equations we have previously used for a freely falling object. That is:

$$y = \frac{gt^2}{2} \quad (1) \quad \text{and} \quad v_y = gt \quad (2)$$

Note that the y-axis is, contrary to the mathematics common use, oriented downwards while x-axis is the usual horizontal direction.

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Improved experiment

A physics experiment and the accompanied measurements should be as simple as possible. But when they cannot give us enough data to explain the phenomenon, one must enhance or redesign the experiment. That is the reason to continue this experiment using a spring-gun (Projectile launcher produced by PASCO) fixed on the with-board pillar (Figure 2). It enables us to shoot a plastic ball repeatedly. Such an enhanced experiment enables us to measure quantities which lead us to the conclusions about the horizontal part of this movement. First, students would not realize, how using this gun could help us to analyse the case of a horizontally thrown object. They say: "The velocity of the ball is very high. How can we measure this movement?"



Fig. 2

A spring gun is fixed on the white-board pillar. Its position in the classroom is more visible on Figure 3.

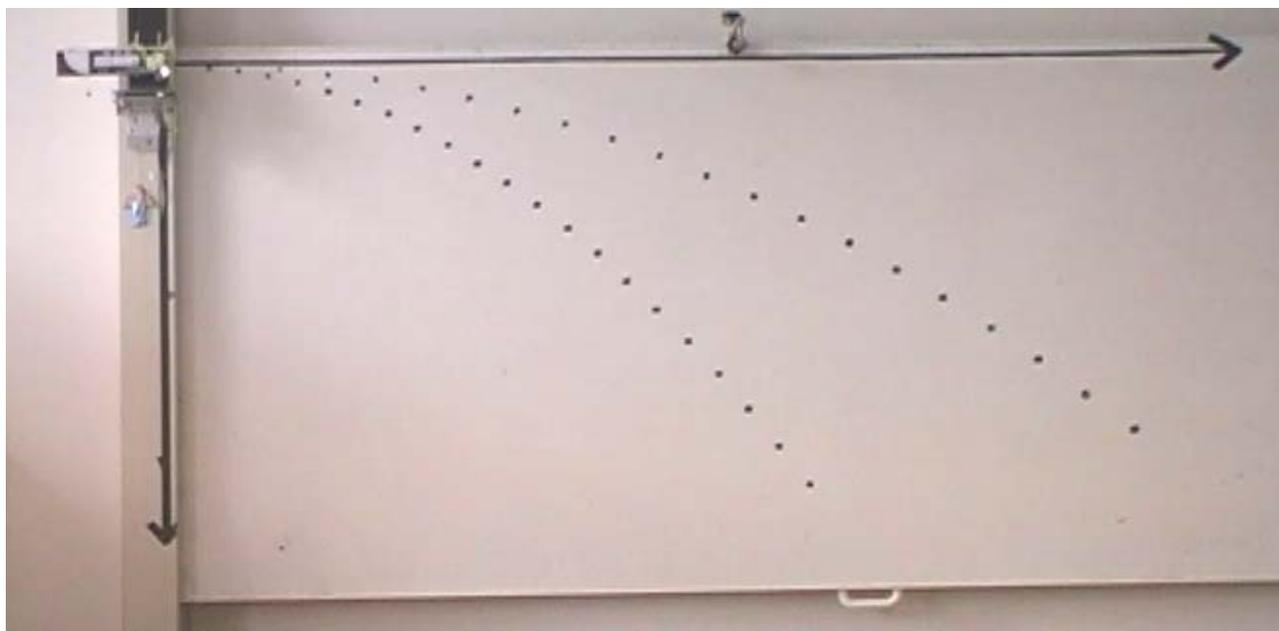


Fig. 3.

This is photography of the whiteboard in the classroom during physics lesson. The dots have been drawn while we projected the horizontal throw in slow motion mode.

The simplest way is to record the experiment using a digital camera. Later we can analyse the slow motion of the ball's path. It is even not difficult to place the movie projector at the right distance from the whiteboard in order to get a 1:1 scale projection. The movie is projected in slow motion; so slow, that the positions of the ball at the time equidistant points can be marked. The gun enables us to shoot with three different initial velocities. The two of them are recorded and projected. The trajectory's marks are shown in Fig. 3.

Some students notice that the distances between marks in horizontal direction are the same when we consider one shoot (with particular velocity). But the majority of students will realize that, when we point it out on the board (Figure 4).

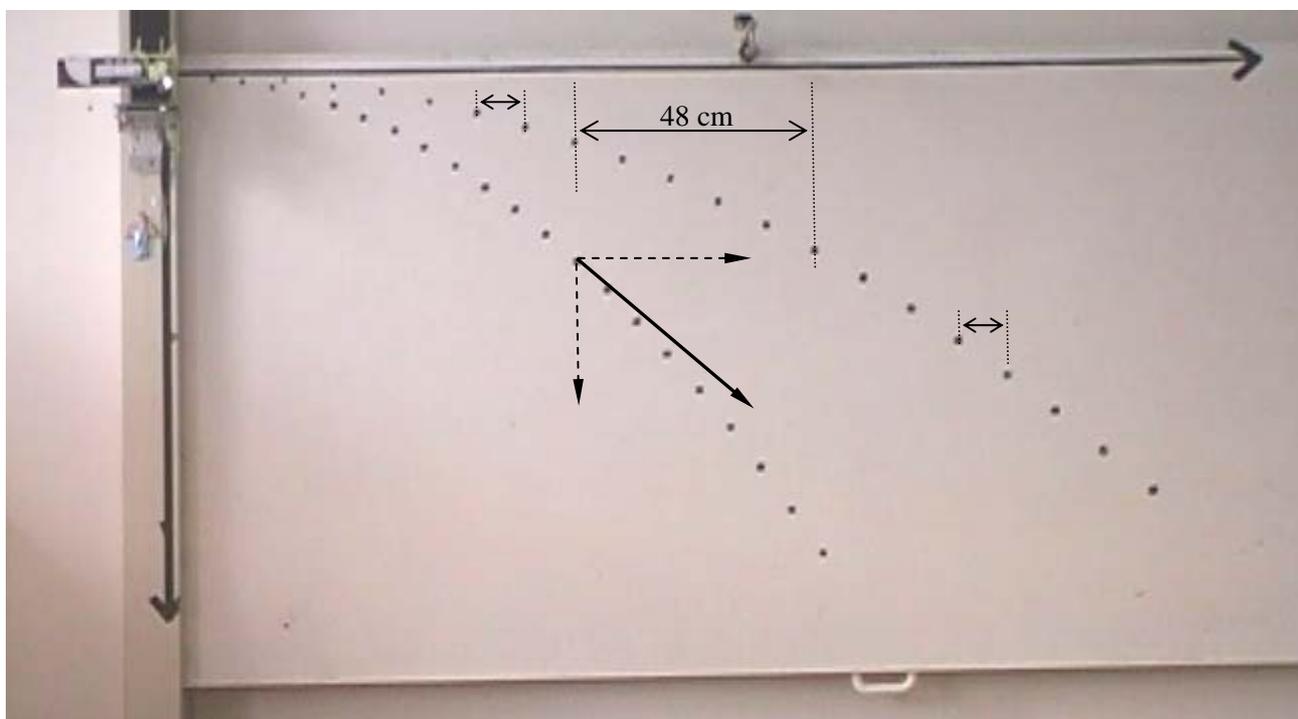


Fig. 4.

As the dots have been projected in real size one can simply measure the distances on the whiteboard and calculate the initial velocity. It is obvious that the horizontal distances are the same during each time interval.

It is obvious (Fig.4) that the horizontal component of velocity is constant. We can write the two equations for the horizontal movement:

$$v_x = v_0 \quad (3)$$

$$x = v_0 t \quad (4)$$

The physics teacher simply measures the distances on the whiteboard and calculates the initial velocity. Furthermore he explains the instantaneous velocity as a vector quantity and

its components. He also carries out “A hunter and a monkey experiment” but this is not so essential for mathematics teacher, so this is not described here.

...TO MATHEMATICS

“Mathematics teacher/ students as investigators”

The mathematics teacher motivates his/her students by bringing the photocopies of (Fig. 3) scenes they will recognize from physics lessons (or he/she continues directly from (in) the physics lesson). It is in their physics notebooks already! He invites the students to investigate the picture mathematically. This investigation will lead them to calculate some important facts about shooting without looking at the data which are collected in the physics notebook. First they remember the equations which are certainly known to mathematics teachers as well. Therefore:

$$y = \frac{gt^2}{2} \quad (5)$$

and

$$x = v_0t \quad (6)$$

The mathematics teacher can proceed by acknowledging that he doesn't know the initial velocity of the ball and also, he does not know the time intervals between two marked positions of the ball. But he claims he could find out the missing data by analyzing the picture.

Question: How can we plot a $y(x)$ graph (expressed by the parametric expressions (5) and (6)) which fits the dots on the whiteboard.

Combining the two equations (t is expressed from the equation (6) and used in the equation (5)) one derives the parabola equation:

$$y = \frac{g\left(\frac{x}{v_0}\right)^2}{2} \quad (7)$$

In order to have a more common form he writes:

$$y = \frac{g}{2v_0^2}x^2 \quad (8)$$

Now the equation resembles the well known central position parabola equation $y = ax^2$. When coefficient a is for example 5, the parabola is very steep and vice versa, when a is small (for example 0,2) the curve is gently rising. We must take into account that now y -axe is oriented downward. The positive a gives us the picture equals to Fig. 4, a negative a using an up oriented y -axe coordinate system.

Question: Does this mathematical observation fit the physics of the real world? In the case of higher velocity the slope is less steep. The upper points (Fig. 3, 4 and 5) correspond to higher velocity. The coefficient in equation 8 is smaller in the case of higher velocity.

To the quantitative analysis

This is only qualitative analysis. Now the mathematics teacher is going to choose one point and measures its coordinates (establishing appropriate coordinate system and units). They are 12,0 cm and 4,3 cm (Figure 5). But what is the scale of this picture? Mathematics teacher admits he entered the physics classroom with the help of cleaning lady a day before (Physics classroom is always locked!). There was nothing at the whiteboard so he just measured the height of the whiteboard. He calculated the ratio between the real objects and the picture: it is 1 : 14. The coordinates of encircled dot are calculated by multiplying the measured values by 14. Therefore they are: 1,68 m and 0,60 m.

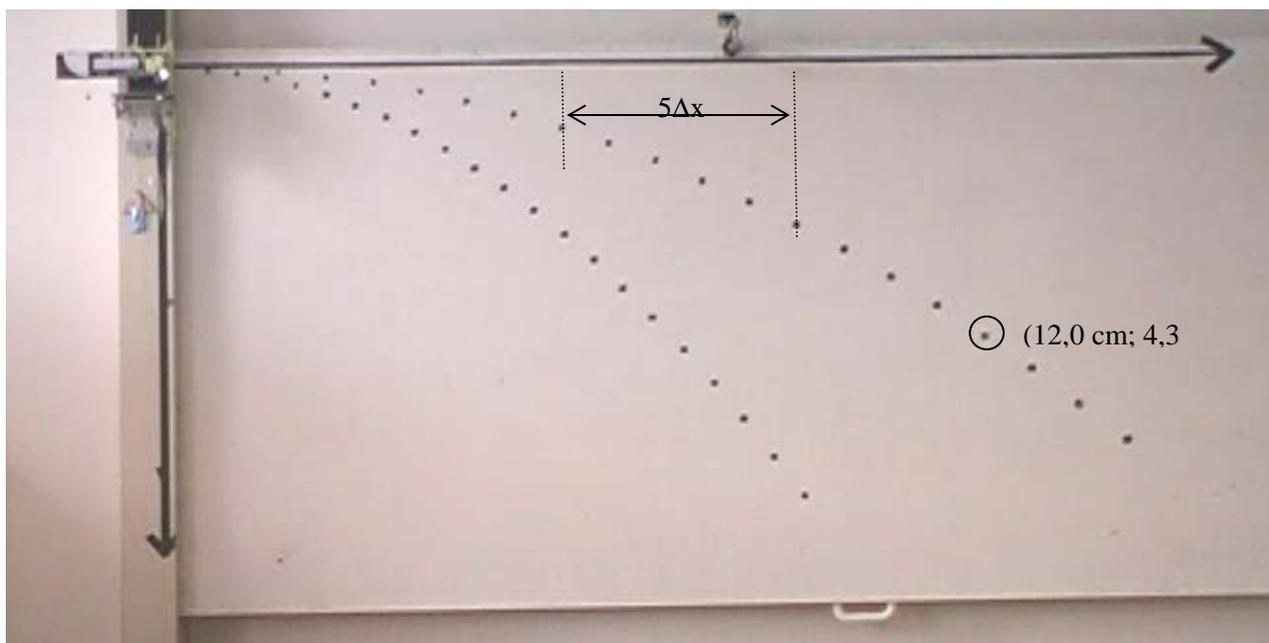


Fig. 5: Mathematics teacher encircled one dot and measured its coordinates. He also measured the displacement in horizontal direction between five consecutive points.

Using the equation (8) we simply calculate the initial velocity. It is 4,8 m/s. The students look up in the physics notebook and find the same result.

But mathematics teacher can find out also the time interval between two dots. Taking into account already calculated initial velocity and measuring the interval $5\Delta x$, he calculates the time interval: 0,0198 s. The inverse value corresponds to the frequency with which the pictures were taken. The frequency is almost 50 s^{-1} which corresponds to the frequency rate of an ordinary digital camcorder. Therefore, the time interval is about 0,020 s.