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COMENIUS

Background

General Didactic Background

The starting point is a cross-curricular approach in the natural sciences, with physics in particular. The objective is, to make high school students experience mathematics as adequate, meaningful and interesting through extra-mathematical contexts. Learning in context is to contribute to an intuitive mathematical understanding. With the help of contexts and methods, derived from the natural sciences, the often noticed gap between formal mathematics and authentic experience shall be closed, and students are, on the other hand, given the opportunity to experience the versatility of mathematical terms.

Natural science contexts offer the opportunity for truly realistic teaching. Concrete physical or biological contexts may stimulate mathematical modelling activities and lead to authentic experiences. Mathematical content and methods are learned in meaningful contexts, and the students' sense of reality can be extended through mathematical insight. Different references to reality produce different models and can therefore also contribute to a contrasting of terminological properties and of a variety of models. The diversity of natural science phenomena allow setting open tasks and thereby working out mathematical problems independently. Mathematical terms, as for example, the concept of the function, can be experienced as modelling activities. The variety of their contextual meanings and of their properties can be grasped in their various connections with reality.

Mathematical Background

The key words in the material presented here are the term function and functional thinking. It is addressed to high school students aged 16 to 18. The lesson plans, suggested here, can be used independently at any time during the school year, or they can be integrated in a course to acquire a better understanding of the term function. The suggested experiments lead to functional connections that cannot readily be described by a functional term. In some parts, only a partial approach is possible, so that in groups of older students and high achiever groups known procedures can be tightened. On the other hand, the a priori unclear graphic progression enforces a discussion about the functional connections and the aspects of functional attribution and in particular of co-variation.
It is suggested to offer the experiments in work stations. The time frame may be set at two double lessons, in which each group should be able to work on a minimum of two stations. At the end of the teaching unit, a presentation phase is suggested, in which each group presents their results of one of their experiments.
The suggested experiments require competences and a sound knowledge of experimental physics on the part of the teacher. The students should be familiar with the physical background or acquire it during their work on the experiments. (Note: In Functional relation 1, there are suggestions of simple experiments.)

The concept of function is one of the most important, but also one of the most difficult mathematical terms. A lot of research has shown that students have only a limited conceptual understanding of the term function. A function is only understood as "something with a x and a y" or something that one represents graphically. The graph of a function is rather recognized the route of the curve than the functional interdependence between two values. In teaching there is, indeed, the danger to primarily reduce the teaching of functions to

drawing of graphs to represent equations. The function concept is much more complex, though. Understanding the function concept means to be familiar with its conceptions in respect of content, its various areas of representation and to switch between them. The complexity of the function concept has been subject of a number of analyses in the last decades. In this context, major work has been done by DeMorois and Tall, Stoye and Fischer/Malle and Swan, identifying different forms of representation, the changes in between and cognitive planes (Cf. bibliography under further information).
To sum it up, <u>three aspects of the function concept in terms of content</u> are distinguished here (Cf. bibliography: Beckmann 2006):

- the correspondence aspect (action: Each element x of a quantity X is allocated an element y of a quantity Y. Here, in a simple case, it is only possible to look at one element x or at one after the other/continuously all x of X).
- the Co-variation aspect (process: If x changes, then the allocated, respectively, the corresponding y also changes. We can always change x discretely or run through the quantity X continuously).
- the object aspect: To understand a function as an object means to understand the function as a whole, i.e. to be familiar with aspects such as simple and continuous allocation, discrete and continuous co-variation in all forms of representation, possible changes and types of changes.

Aspects of representation are situations (images, verbal descriptions), tables, graphs, algebraic expressions / terms.

The function concept is most comprehensively presented in Hoefer's "House of functional thinking" (figure 1). It considers and clarifies all aspects of the function concept and all possibilities of transfers. It also allows the differentiation between various possibilities of the same transformation, i.e. if the graphic transformation of a term is done point by point or under dynamic points of view (figure 1).

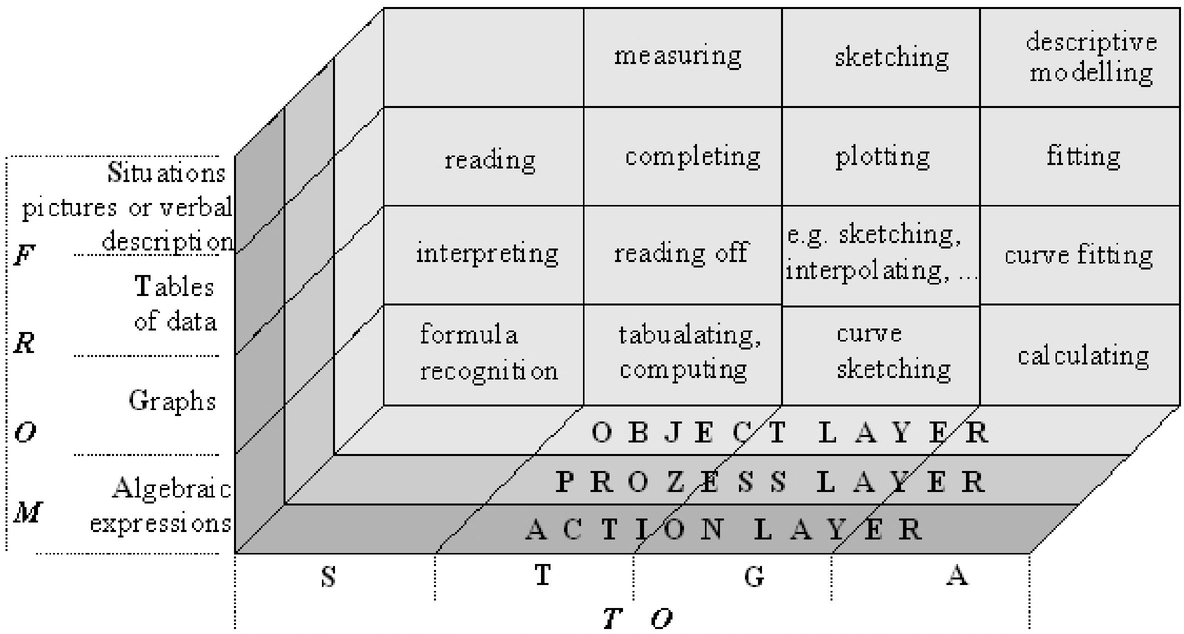

figure 1: House of functional thinking (Cf. bibliography Hoefer 2006, 2008)

The idea of teaching implementation

The basic idea of the teaching sequence suggested here, is to support students from all types of schools through experimental activities, references to reality and natural science contexts in acquiring an adequate understanding of functions. Experiments are a good way realizing this, since the experimental steps correspond to aspects of the concept of functions. In carrying out the experiments, the aspects of the functional concept in terms of content are experienced through activities. Different areas of representation and the changes in between are stimulated through different modelling activities. The often neglected and little trained competences to verbalize, respectively, to interpret graphs, are stimulated by the references to reality and by the concrete values used in the experiments (Cf. extensive explanations in Beckmann 2006). Through this, in particular, the students are given the opportunity to understand functional contexts and discuss them. The aspect of co-variation becomes an authentic experience.

The stimulus for experimental activities will be realized at the various stations. The students are first of all reminded of their own every-day experiences and practical situations through a real-life impulse and are stimulated to discuss changes and formulate hypotheses. Checking the respective hypothesis motivates experimentation, leading to a functional connection. This connection, usually first collected in a table, is then researched further, for example, graphically. In this, the emphasis is on the verbal discussion and the (at least final) reference to the students' every-day life. Final presentations by each group in a class have proved to be very successful for this.

To make sure that the functional relationship is not obvious from the start but has to be worked out, the suggestions are not limited to one type of function (such as linear functions) but address various functional relationships. In addition, the special thing about the experiments suggested here, is the fact that the functional relationship is not immediately clear but only partial results from the experiments. This forces students to come to terms with the content of the functional relationship. If having strong achiever groups, one can try to find the corresponding functional term, which, however, does not work easily for all examples and can possibly only be done through approximation/fit- procedures. In any case, what is even more important, is the verbal and contextual discussion of the changeability and the dependence on other factors of the values observed.
The experiments suggested under "Teaching material", are suitable for students at the beginning of secondary II (respectively at the end of secondary I). The experiments require competences and knowledge of experimental physics on the part of the teacher. The students should already be familiar with the physical background or acquire it while working on the experiments. The experiments, collected in the sequence, are designed to teach students to come to terms with the verbal and the content aspects of the changes and the dependences of the values observed. The basis are a variety of functions, which, however, are not the usual simple types of functions taught in school but rather unfamiliar functions, respectively, functional relationships, which only originate in the process of the experiment. The experiments require teachers with some physical background and partly competences in experimental physics. A cross-curricular approach, that considers the physical terminology in addition to the mathematical interest in functional dependency, is highly desirable here, and in parts also necessary. (Reference: In other teaching examples, experiments with simpler content are suggested).

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