The **ScienceMath**-project: **Functional relations 2**
Idea: Astrid Beckmann,
University of Education Schwaebisch Gmuend, Germany

Education and Culture
Lifelong learning programme
COMENIUS

Teaching Material

Suggested lesson plan

Introduction	The teacher introduces the lesson. Possible topics: measuring errors, drawing functional graphs/curves – → literature, resp. physical background
Stations	The experiments are offered in stations and are to be carried out independently and in their own responsibility (worksheets offer stimulation) 
Final session in class	Each group presents the results of their group work at the station

Material needed and Experiments (see next pages)

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Station 1
Experiment Electric car

Variables	Distance and time	
Interrelation	Linear / proportional	
Material	Car with electric motor, measuring tape (minimum 2 m), stop watches	
Procedure	Measuring time for certain distances of the car	
Cross-curricular background	The car moves along a level track. This straight steady movement has the following characteristic: the same distances are covered in the same time, i.e. the distance s and the time t are proportional: $\frac{s}{t} = const.$ The constant here describes the unchanged value, the velocity v. The unit for the velocity is $\frac{m}{s}$, $\frac{metre}{second}$.	
Relation to reality	Driving in a car	

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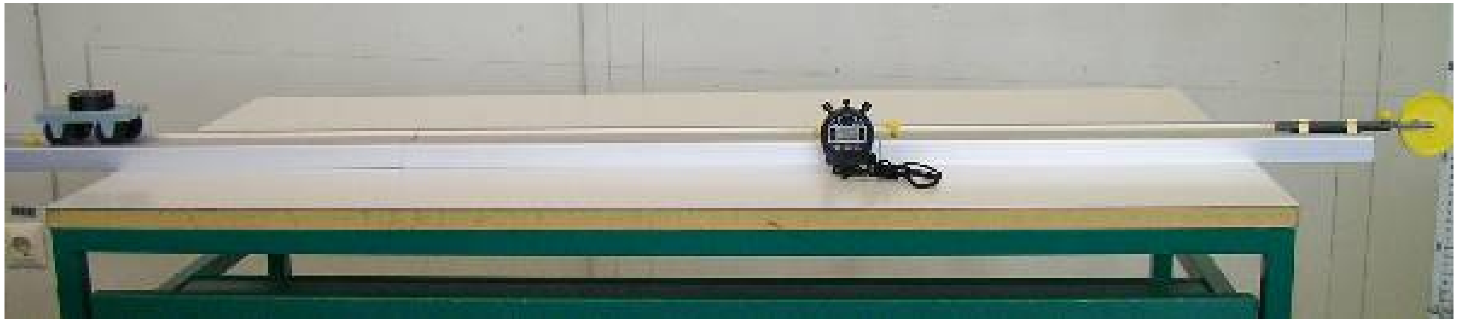
Station 2:
Experiment: car race

Variables	Distance and Time
Interrelation	quadratic
Material	Smooth-running car (weight 50 g, plus 200 g load) with mass of 50 g fixed by thread to a pin because of acceleration. Thread running over a wheel. Guide rail (minimum 1.5 m – if available), tape measure, stop watch
	
Procedure	The car drives – evenly accelerated by the falling piece of mass. The times for various distances are recorded by the stop watch.
Cross-curricular background	The car moves along the track in even acceleration. The acceleration is achieved by the weight of the piece of mass attached to the thread. The piece of mass is accelerated through gravity and so is the car attached to the thread. This uniformly accelerated movement has the following characteristics: $s \sim t^2$ also $\frac{s}{t^2} = \text{const.}$ The constant corresponds to half the acceleration a, measured in m/s². The following applies: $s = \frac{1}{2}at^2$ (distance-time-law of accelerated movements)
Relation of reality	Accelerated movements, car journey, car race

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Station 3:
Experiment falling movement

Variables	Distance and duration of fall	
Interrelation	quadratic	
Material	ball (tennis ball oder juggler´s ball), tape measure, stop watches, hall way or building with several floors, where the ball can be dropped on the inside or outside and the distance of the fall can be measured.	
Procedure	Various positions, from which the ball is to be dropped, are fixed. The respective distances of the falls are measured with the help of the tape measure. After this, the ball is dropped from the positions and the time of the respective fall is recorded.	
Cross-curricular background	On Earth, a falling object drops at an evenly accelerated speed of $g = 9{,}81\frac{m}{s^2}$ (Acceleration in central Europe, disregarding aerodynamic drag). This evenly accelerated movement has the following characteristics: $s \sim t^2$ *therefore* $\frac{s}{t^2} = const.$ The constant corresponds to half the earth's acceleration g. The following applies: $s = \frac{1}{2}gt^2$ (time-distance-law of the free fall)	
Relation to reality	“Freefall-tower“ in a theme park, falling objects in every-day life.	

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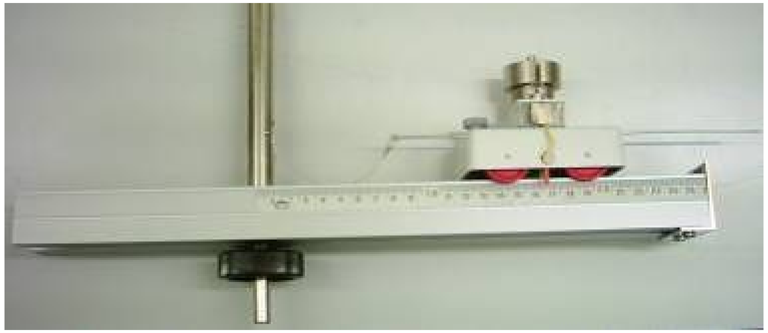
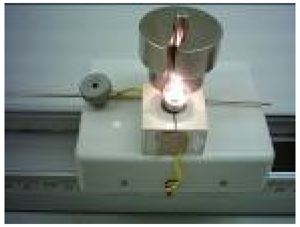
Station 4
Rotational movement

Variables	angular speed and centripetal force	
Interrelation	quadratic	
Material	Electric motor with drive, revolving track (mounted on the electric motor), force metre (up to about 0.5 N), car, moving on the track and connected to the force metre. Pay attention that mass and distance to the turning axle have to be kept constant, so that the centripetal force always has to be read at the identical position of the car.* 	 Addition: *This can be made clear by an optical or an acoustical signal. 
Procedure	By switching on the electric motor, the arm starts to rotate. The centripetal force pushes the car towards the outer edge of the rotating arm. The centripetal force of the fixed angular speed is read at the defined point by the force metre. Here, the angular speed corresponds to the speed of the arm, rotated by the electric motor, and the centripetal force results from the deflection of the spring resistance (caused by the car's movement on the rotating arm; due to the centripetal force the car is driven outwards.)	

Cross-curricular background	Through the rotation, the car is moved along a circular track. A force, that is perpendicular to that movement, is directed at the car (as a rotating mass point). If that force was not effective, the car would (theoretically) fly off in a straight line. The circular movement is uniform, i.e. equal angles $\Delta\varphi$ are passed over in equal times Δt: The angular speed $\omega = \frac{\Delta\varphi}{\Delta t}$ is constant. The car, however, is constantly accelerated since the direction of the speed vector changes continuously. The centripetal acceleration is $\omega^2 \cdot r$ (r is the distance to the turning axle). The effective force is thus: $F_Z = m\omega^2 r$
Relation to reality	Rotational movements, as with carousels at theme parks and fun fairs

(experiment rotational movement)

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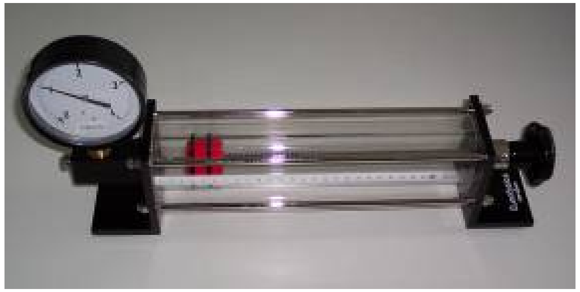
Station 5:
Experiment: Pressure

Variables	pressure and volume of gases
Interrelation	inversely proportional
Material	A Boyle-Mariotte instrument, i.e. a gas-filled glass cylinder. By pushing in the seal (with the help of a rotary knob at the end) the volume of the gas can be reduced (can be read at a measuring scale). The pressure change is indicated directly at the integrated manometer. Replace as necessary by a bicycle pump for a start (cf. below) 
Procedure	Using the rotary knob, the volume is changed and the resulting pressure can be read at the manometer.
Cross-curricular background	Assuming an ideal gas, pressure and volume behave anti-proportionally: pV=constant. (Under normal conditions, this is best met by H_2 and He. The normal gas should not be too dense and not too cold.) At a given temperature, the constant only depends on the number of molecules the gas volume contains. This law is named after their discoverers the Boyle-Mariotte-Law (Robert Boyle: English physicist and chemist 1627-1691, Edme Mariotte: French physicist 1620-1648). One can find this out by oneself, if one pushes together a closed bicycle pump (the volume is reduced and the pressure increases). 
Relation to reality	Pressure-volume changes in gases, closed bicycle pump

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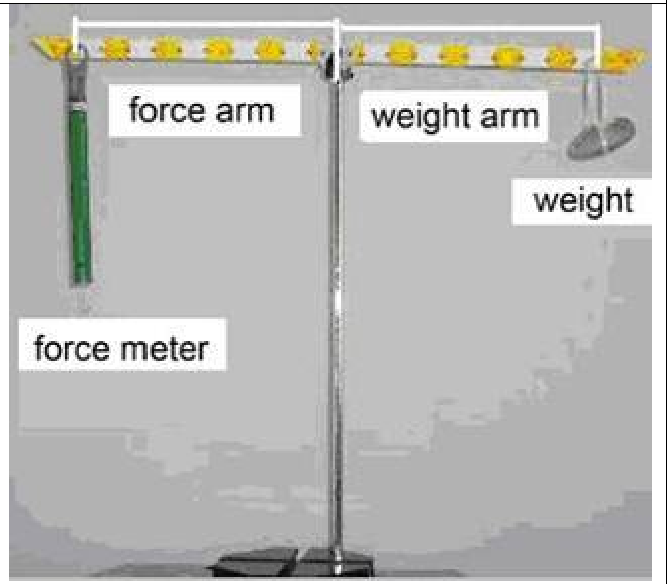
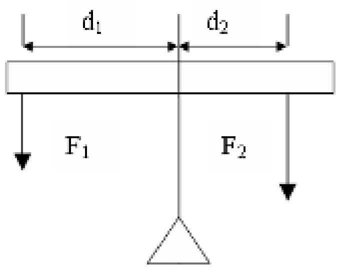
Station 6:
Experiment Lever 2

Variables	Force and lever arm	
Interrelation	Inversely proportional	
Material	Stand with lever rod (length 0.5 m), Force metre (max. force 10 N), Weight (about 100g, f. e. a stone)	
Procedure	The weight (stone) is attached to a lever rod at a fixed distance to the axle, where it stays during the entire experiment (weight/load and load arm are thus constant). The force meter is hung on the other side of the axle and the lever is kept in balance. The lever arm of a force (power arm), i.e. the distance between the lever axle and the force metre, is changed in that the force metre is hung in different places on the lever rod. The force that is needed to keep the lever in balance is measured in dependence of the power arm.	
Cross-curricular background	Simplified, one can picture a lever as a beam with a pivotal axle. Forces can be effective on both sides of the axle. The distance d between the pivotal axle and the force F is called leverage (see figure).	

	 Annotation: If one distinguishes between force on the one side and weight/load on the other, one speaks of power arm and load arm. A lever is exactly in balance if the product of the values of the force and the arm of the lever are the same on both sides of the pivotal axle. $F_1 \cdot d_1 = F_2 \cdot d_2$ (resp. force times power arm = load times load arm) The following functional relations can be derived from the conditions of balance: *Proportionality* between force and load arm $\frac{F_1}{d_2} = const.$ (power arm and load are constant) *Anti-proportionality* between force and power arm $F_1 \cdot d_1 = const.$ A constant load F_2 is attached to the lever at a fixed distance d_2.
Relation to Reality	Seesaw in a playground, lever on a crane, pair of pliers, bicycle etc. Transportation of heavy objects, bag and walking stick of a journeyman.

(Experiment lever 2)

Station 7:
Experiment Tunnel

Variables	distance to light source and intensity of light
Connection	“further” square reduction
Material	Light metre (Lux metre), several cardboard tubes of the same diameter, window with daylight (window pane serves as light source)  
Procedure	The cardboard tubes are held closely to the window pane. On the other side, the light sensor of the lux-metre is held closely to the pane. The intensity of the light can be read on the display. 
Cross-curricular Background	A lightsource (sun, lamp etc.) sends light of a special quantity. A recipient (eye, photo-diode etc.) “feels” a special brightness. The lux-meter measures the brightness in lux. Lux-meters “feel” similar like the human eye. They don’t measure the energy of the light; they measure how bright the lighting seems to be to an eye. Light of same energy but different colours

	seem to have not the same brightness. The measurement is realized by silicium-diodes, which are connected in locking direction. Action of light causes an electric current, which is a measure for the brightness. The brightness or intensity of lighting is the ratio between light current on a plane and area of the plane. 680 lux relates to an area of 1 m² lightened by monochromatic yellow-green light (550 nm) of 1 Watt. 0,1 Lux relates to red light (750 nm) under the same conditions. Examples of light intensities: - sunny summer day outside: about 100000 lux - covered sky in summer: about 20000 lux - dimmed winter day: about 3000 lux - good street lamp: about 40 lux - night with full moon: about 0,25 lux. Good lightening helps to avoid accidents. For work there are regulations which demands 100 to 250 lux and 1000 lux for precision work. The brightness/ intensity of light depend also on the distance of the light source. The intensity decreases quadratically with the distance.
Relation to Reality	Ride into a tunnel (without seeing the end), distance to a lamp (street light, desk lamp)

(Experiment tunnel)

Worksheets

Worksheets for independent work and for selected work stations can be found in the e-book ”Experiments for acquiring Functional Terms” by Astrid Beckmann, Aulis-Verlag, Cologne 2006. The following impulses were also taken from that book.

Impulses for the Workstations

Impulse — Electric Car

Imagine you are sitting in this car, and this car

1. approaches a traffic light.
2. turns a corner.
3. is driven for a long time along a straight road.

Describe the different movements of the car.
Discuss this in your group.

General Task
Describe the relation between distance and time needed. Check: Does the relation confirm your assumption above? Describe the special qualities of that connection.

Impulses — Car Race

In drag-racing, the drivers race at top speed from the beginning. The driver tries to cover a straight distance as fast as possible.
Which conditions can be changed in this race? (Consider the model also.)
Which variables depend on each other?
If you change one of the values, how - do you assume – does the other value change?

www.pixelquelle.de ID34759, Fotograph: NeoNow

Discuss this in your group.

The drag racing track is reconstructed in a simplified form on a desk.
Get familiar with its structure:

- The track is straight.
- The car is "driven" via a string, connected to a weight and hung over a reel. How does that work?

Discuss this in your group.
If you change the distance, how does the time needed change?

General Task
Describe the relation between the distance and the time needed. Check: Does the relation confirm your assumption above? Describe the special qualities of that connection.

Impulse — Free Fall

In theme parks or fun fairs, you often find a special attraction, the “Freefall Tower”. It is usually a slim, 50 m high steel construction, where you are first given a lift up and then allowed to drop down again. Have you done this already? What kind of a feeling was it?

The falling distance in these towers can be different lengths. Where, do you think, is the basic difference between a long and a short fall?

Discuss this in detail in your group.

www.pxelquelle.de ID99300, fotograf: anjume

Assumption:
If you fall 20 m in about 2 s, how long does it take for 40 m?
Tick: about 3 s ☐ about 4 s ☐ about 5 s ☐

What kind of connection, do you assume, exists between the falling distance and the time of the fall? *Discuss this in your group.*

General Task
Describe the relation between the falling distance and the time during the free fall.
Check: Does the relation confirm your assumption above?
Describe the special qualities of that connection.

Impulses — Rotary Motion

We are sure you know carousels, where you are moved very fast around a circular track.
You can see an example of this in the picture. Usually, the rotary movement starts slowly and then accelerates very fast.
Compare the fast and the slow movement.

Discuss in your group, what are you feeling in such situations and how do you react to these.

In the experiment, the carousel is rebuilt in a simple model.
Get familiar with its parts.
What, do you think, is the connection between the angular speed and the centripetal force?
Make a graph that illustrates the relation.

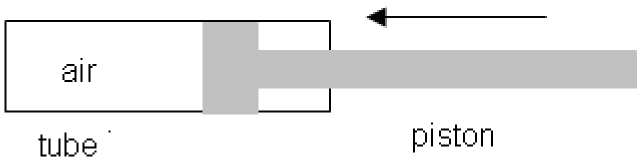
General Task
Describe the relation between angular speed and centripetal force. Check: Does the relation confirm your assumption above? Describe the special qualities of that connection.

Impulses — Pressure

There is a bicycle pump on the desk.

Pull out the piston. Close the vent of the pump and pump.
What do you notice?
Discuss this in your group.

Look at the schematic drawing of a (closed) bicycle pump.



Now, describe in more detail:
What happens when the piston is pushed in? What changes in the process?

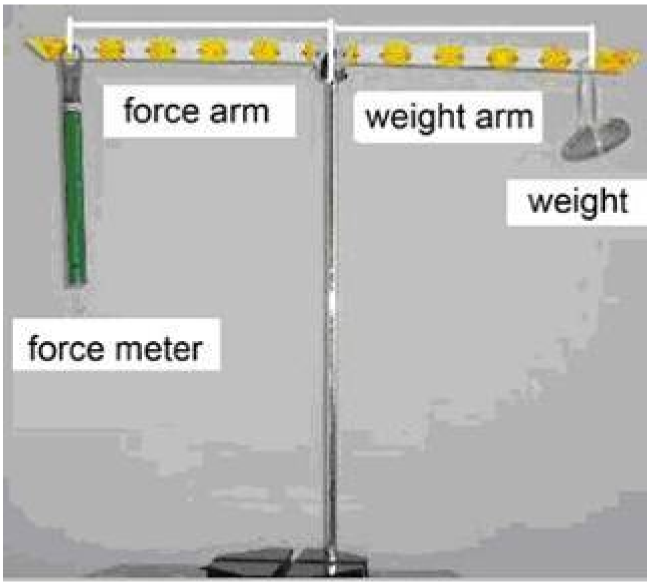
General Task
Describe the relation between the air volume and the air pressure. Check: Does the relation confirm your assumption above? Describe the special qualities of that connection.

Impulses — Lever 2

In playgrounds, you can sometimes find seesaws. I am sure you have played on one, too. Maybe, you have also tried to keep the seesaw in perfect balance with your partner.
Imagine that two friends of yours are standing by the edge of the playground waiting to have their turn. Anton is heavier than you, Bert is lighter.
Assumption: Does Anton have to place himself closer to or further away from the pivotal axle than you?
Does Bert have to place himself closer to or further away from the pivotal axle than you?

Discuss these questions in your group.

The seesaw is rebuilt in an experiment.
In the picture below, the elements are identified by names.



You can hold the force metre so that the lever (the see-saw) is in balance.
You can read the amount of force used at the metre.
(How does that work?)

Assume: Do you have to use more force or less, if you fix the force meter closer to the pivotal axle?

Do you have to use more force or less, if the force meter is hung further away from the pivotal axle?

Discuss this in your group.

General Task
Describe the relation between force and lever arm. Check: Does the relation confirm your answer to the question above? Describe the special qualities of the connection.

Impulses — Tunnel

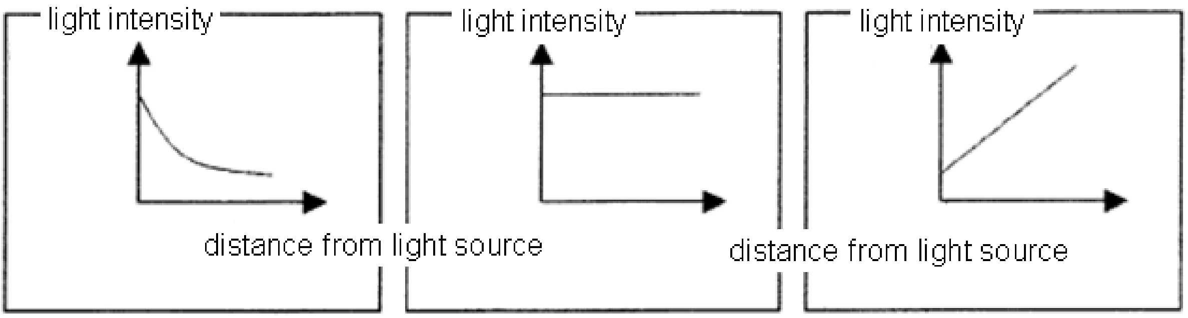
Imagine you are going or driving through a long tunnel. You cannot see the other end of it.
How does the light intensity in the tunnel change, without considering the car lights?

Discuss this in your group.

Now, take a cardboard tube. Hold it against the window (the window represents the light source – sunlight). Observe the intensity of the light.
Now, take a cardboard tube of a different length and hold that against the window. Observe the intensity of the light again.
Compare.

Discuss it in your group.

Which diagram shows the most likely presentation?
Mark it and compare, at the end of the experiment, if your assumption was correct.



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General Task
Describe the relation between the distance to the light source and the light intensity.
Check: Does the relation confirm your answer to the question above?
Describe the special qualities of the connection.