



Background

General Didactic Background

The starting point is a cross-curricular approach with the natural sciences, especially physics. The objective is, to make high school students experience mathematics as adequate, meaningful and interesting through extra-mathematical contexts. Learning in context is to contribute to an intuitive mathematical understanding. With the help of contexts and methods derived from the natural sciences, the often noticed gap between formal mathematics and authentic experience shall to be closed, and students are, on the other hand, given the opportunity to experience the versatility of mathematical terms.

The natural science contexts offer the opportunity for truly realistic teaching. Concrete physical or biological contexts may stimulate mathematical modelling activities and lead to authentic experiences. Mathematical content and methods are learned in meaningful contexts, and the students' sense of reality can be extended through mathematical insight. Different references to reality produce different models and can therefore also contribute to a contrasting of terminological properties and of a variety of models. The diversity of natural science phenomena allow setting open tasks and thereby working out mathematical problems independently. Mathematical terms, as for example the concept of function, can be experienced as modelling activities. The variety of their contextual meanings and of their properties can be grasped in their various connections with reality.

Mathematical Background

The concept of function is one of the most important, but also one of the most difficult mathematical terms. A lot of research has shown that students have only a limited conceptual understanding of the term function. A function is only understood as "something with a x and a y " or something that one represents graphically. The graph of a function is rather recognized the route of the curve than the functional interdependence between two values. In teaching there is, indeed, the danger to primarily reduce the teaching of functions to drawing of graphs to represent equations. The function concept is much more complex, though. Understanding the function concept means to be familiar with its conceptions in respect of content, its various areas of representation and to switch between them. The complexity of the function concept has been subject of a number of analyses in the last decades. In this context, major work has been done by DeMorois and Tall, Stoye and Fischer/Malle and Swan, identifying different forms of representation, the changes in between and cognitive planes (Cf. bibliography under further information).

To sum it up, three aspects of the function concept in terms of content are distinguished here (Cf. bibliography: Beckmann 2006):

- the correspondence aspect (action: Each element x of a quantity X is allocated an element y of a quantity Y . Here, in a simple case, it is only possible to look at one element x or at one after the other/continuously all x of X).
- the Co-variation aspect (process: If x changes, then the allocated, respectively, the corresponding y also changes. We can always change x discretely or run through the quantity X continuously).
- the object aspect: To understand a function as an object means to understand the function as a whole, i.e. to be familiar with aspects such as simple and continuous

allocation, discrete and continuous co-variation in all forms of representation, possible changes and types of changes.

Aspects of representation are situations (images, verbal descriptions), tables, graphs, algebraic expressions / terms.

The function concept is most comprehensively presented in Hoefler's "House of functional thinking" (figure 1). It considers and clarifies all aspects of the function concept and all possibilities of transfers. It also allows the differentiation between various possibilities of the same transformation, i.e. if the graphic transformation of a term is done point by point or under dynamic points of view (figure 1).

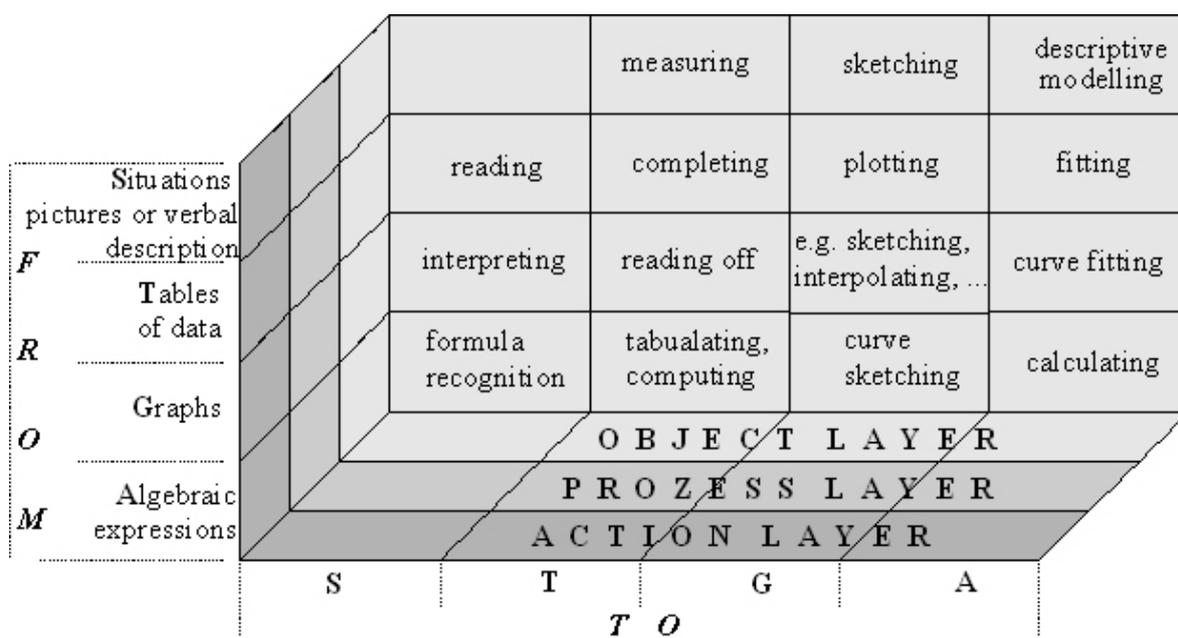


figure 1: House of functional thinking (Cf. bibliography Hoefler 2006, 2008)

The idea of teaching implementation

The basic idea of the sequence suggested here is to support students from all types of schools through experimental activities, references to reality and natural science contexts in acquiring an adequate understanding of functions. Experiments are a good way of realizing this, since the experimental steps correspond to aspects of the concept of functions. In carrying out the experiments, the content aspects of the functional concept are experienced through activities. Different areas of representation and the changes in between are stimulated through different modelling activities. The often neglected and little trained competences to verbalize, respectively, to interpret graphs, are stimulated by the references to reality and by the concrete values used in the experiments (Cf. extensive explanations in Beckmann 2006). Through this, in particular, the students are given the opportunity to understand functional contexts and discuss them. The co-variation aspect becomes an authentic experience.

The stimulus for experimental activities will be done at various stations. The students are first of all reminded of their own every day experiences and practical situations through a real life impulse and are stimulated to discuss changes and formulate hypotheses. Check-

ing the respective hypothesis motivates experimentation, leading to a functional relation. This relation, usually first collected in a table, is then researched further, for example graphically. In this, the emphasis is on the verbal discussion and the (at least final) reference to the students' every day life. Final presentations by each working group in a class have proved to be very successful for this.

To make sure that the functional relation is not obvious from the start, but has to be worked out, the suggestions are not limited to one type of function (such as linear functions) but address various functional relations. In strong achiever groups, one can try to find the corresponding functional term, which, however, does not work easily for all examples. In any case, what is even more important is the verbal and contextual discussion of the values observed, including their changeability and their dependence on other factors.

The examples, suggested here, are suited to students at the end of secondary I, respectively, early secondary II. In some parts, the experiments require competences in experimental physics on the part of the teacher. A cross-curricular approach that, apart from the mathematical interest in functional dependencies, which also takes into account the physical terminology, is very desirable here and presumably in part necessary.