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Teaching material

Regarding functional relations there exist a lot of experiments (see Literature → Beckmann 2006). The selection can be made in different ways and according to class or capacity level. The following examples relate to an implementation that may be realized easily in the daily routine in school and which takes place in maths in the classroom – which means not necessarily in a physics room or any room with extensive collections and access to complex tests set-ups. Material and correlations are therefore chosen simply[1]. In order that the functional contexts are not obvious and have to be investigated the suggestions do not refer only to one functional type (e.g. linear function) but they address to various functional contexts (see Literature → Beckmann 2006a). Within efficient learning groups one can try to work out the corresponding concept. However more important is the verbal and textual analysis of the modification- and interdependency manner of the regarded sizes in all cases.

Possible course

Introduction	Teacher introduces into the work, Possible themes: measuring errors, drawing the line of best fit etc. → Literature
Station work	The experiments are arranged in stations and should be applied in self-dependend work (e.g. use worksheets) 
Plenary session	Every group presents the results received during the work at one station

The experiments (material and description see next pages)

This project has been funded with support from the European Commission. This publication reflects the views only of the author(s), and the Commission cannot be held responsible for any use which may be made of the information contained therein.

[1] The here mentioned suggestions for experiments are a possible selection. It is quoted partly from (Beckmann 2006) where many suggestions can be found.

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Station 1:
Experiment *Electric Car*

Dependent quantities	Distance and time
Dependence	linear
Material	Electric Car, measuring cord (at least 2 m), stop watches 
Performance	Measuring time for certain distances of the car
Interdisciplinary background	The car moves with constant velocity straight on. This linear constant movement has the following property: In equal times equal distances are covered, which means that distance and time are proportional: $\frac{s}{t}$ = constant. The constant value describes the here unchanged size, the velocity v. The unit for velocity is $\frac{m}{s}$ (meter per seconds).
Reference to reality	Driving a Car

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Station 2: Experiment *Drop*

Dependent quantities	Volume and number of drops	
Dependence	proportional	
Material	Tripod with mounting for separating funnel, Measuring cylinder with millilitre scale, Water	
Performance	Separating funnel is filled with water. The valve is opened so that the water drops slowly (drop by drop) into the measuring cylinder which is below. The number of drops is counted. The volume of the water is measured by the scale of the measuring cylinder.	
Proportional factor	The quotient of volume and number of drops is constant. The constant corresponds to the volume of one drop.	
Interdisciplinary background	With dropping of the water out of the separating funnel the volume of the water in the measuring cylinder increases. If we can manage to let the water drop down consistently in the same volume, the volume changes proportional to the number of drops. $V \sim n$ (V = Volume of the water in the measuring cylinder, n = number of drops) $\frac{V}{n} = \text{konst.}$ The constant corresponds to the volume per *1* drop. Concerning the term drop: in general a drop is a liquid formation which takes the form of a globe because of the surface tension. The so called drop form is adopted from falling drops which is a consequence of the least air resistance. In the experiment the term drop is used for the quantity of water that is dropping down.	
Reference to reality	Water wasted by a dropping tap, water supply on earth	

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Station 3
Experiment *Free Fall*

Dependent quantities	Distance and time	
Dependence	quadratic	
Material	Ball (Tennis), measuring cord, stop watches, Stairways in school, where the ball can be dropped and the falling distances can be measured.	
Performance	At first various positions are marked from where the ball will be dropped. The particular drop lines are measured with the tape. Afterwards the ball is dropped from the marked positions and the particular falling time is measured.	
Interdisciplinary background	On earth a falling object moves consistently accelerated with $g = 9{,}81\frac{m}{s^2}$ (earth acceleration in Central Europe – air resistance neglected). This consistent accelerated movement has the following property: $s \sim t^2 \quad \textit{therefore} \quad \frac{s}{t^2} = \textit{const}.$ The constant corresponds to half earth acceleration g. It is: $s = \frac{1}{2}gt^2$ (way-time-law of drop movement) <u>Information</u>: The quotient s/t is not constant in this case. It increases with time. It corresponds to half of the actual velocity of the ball at a point of time t. The quotient is average speed. As the velocity increases from value 0, the average is one half of the velocity at the end of the time interval.	
Reference to reality	“Free-fall”-Tower in amusement parks, falling things in everyday life	

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Station 4
Experiment *Cylinder*

Dependent quantities	Radius and volume of the cylinder
Dependence	quadratic
Material	Cylindrical cans or tubes of equal height but different radius of base, Ruler to measure the radius, Sand to fill the cylinders (best: bird sand), Measuring jug, If necessary funnel and even pad  
Performance	The cans/ tubes are filled with sand and the volume is measured with the measuring jug in dependence of the radius of the base of the cylinder
Background	The volume of a cylinder is calculated by area of the base times height. The base of a cylinder is a circle. Area of the circle: $A = \pi r^2$ with r = radius of the circle and $\pi = 3{,}14159$. Out of this result for the volume of the cylinder: $V = \pi r^2 h$. With constant h it is: $V \sim r^2$ res. $\frac{V}{r^2} = const.$
Reference to reality	Cylindrical shape is a convenient package shape for food, medicine, food supplement etc.

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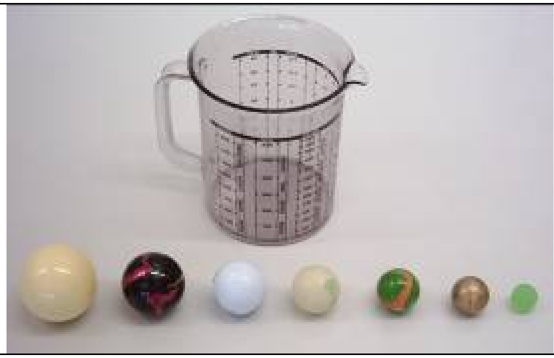
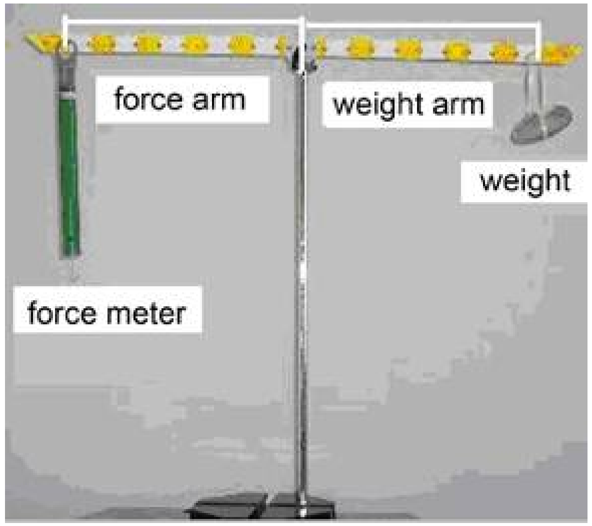
Station 5
Experiment *Globe*

Dependent quantities	Radius (of globes) and volume (of the displaced liquid)
Dependence	Cubic
Material	A measuring jug with water, Various globes of different radius, A slide gauge 
Performance	The measuring jug is filled with water and the volume of the water is determined. The radii of the globes are measured by the slide gauge. The globes are plunged one after the other into the water and the new volume is measured. The volume of the displaced liquid can be calculated as difference and in dependence of the radius of the plunged globe.
Background	Any globe that is totally plunged into the water displaces its own volume. The volume of a ball is calculated through $V = \frac{4}{3}\pi \cdot r^3$ (V = volume, r = radius of the globe).
Reference to reality	Displacing of water by objects, e. g. when bathing (Archimedes in the bath), story of the Frog King.

Station 6
Experiment *lever 2*

Dependent quantities	Force and weight arm	
Dependence	Inverse proportional	
Material	Stand with lever bar (length 0,5 m), Force meter (max. 10 N), Hanging weight (app. 100 g, e.g. a stone)	
Performance	The force meter is mounted to a certain place at the lever bar, where it remains during entire experiment (therefore force arm is constant). weight arm (distance between force and spin axis of the lever) is changed by hanging the weight to different places of the lever bar. The force that is needed to hold the lever in balance is measured in dependence of the weight arm.	
Interdisciplinary background	*See next page*	

<table>
<tr>
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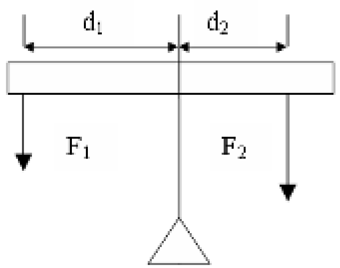
A lever can be imagined as a beam with a rotation axis. On both sides of the rotation axis forces may act. The distance d_1 between rotation axis and force F_1 is called lever arm (according to the figure below)



Remark: If we distinct between force on the one side and weight on the other side we talk about force arm and weight arm

A lever is exactly in balance if the products of the amount of the force and lever arm are equal on both sides of the rotation axis.

$F_1 \cdot d_1 = F_2 \cdot d_2$ (res: force by force arm = weight by weight arm)

The following functional coherences may be deduced from the balance condition.
proportionality between force and weight arm $\frac{F_1}{d_2} = \text{const.}$ (force arm and weight are constant)
inverse proportionality between force and force arm $F_1 \cdot d_1 = \text{const.}$
A constant weight F_2 is mounted to the lever in a certain distance d_2.
</td>
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<tr>
<td>Reference to reality</td>
<td>Seesaw on a playground, lever at a crane, claw, bicycle etc., transport of heavy subjects (bag and hiking sticks of the travelling journeymen)</td>
</tr>
</table>

(Exeperiment lever 2)

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Station 7
Experiment *tunnel*

Dependent quantities	Distance of a light source and brightness	
Dependence	Inverse “quadratic” (see below *interdisciplinary background*)	
Material	Measuring instrument for brightness measurements (Lux-meter), cardboard tubes of different length and same diameter, window with day-light (place of window is place of light-source)	
Performance	The cardboard tubes are hold with one side at the window. At the other side the sensor of the lux-meter is fixed. The brightness is directly shown on the display.	
Interdisciplinary background	*See next page*	

	A lightsource (sun, lamp etc.) sends light of a special quantity. A recipient (eye, photo-diode etc.) “feels” a special brightness. The lux-meter measures the brightness in lux. Lux-meters “feel” similar like the human eye. They don’t measure the energy of the light; they measure how bright the lighting seems to be to an eye. Light of same energy but different colours seem to have not the same brightness. The measurement is realized by silicium-diodes, that are connected in locking direction. Action of light causes an electric current, which is a measure for the brightness. The brightness or intensity of lighting is the ratio between light current on a plane and area of the plane. 680 lux relates to an area of 1 m² lightened by monochromatic yellow-green light (550 nm) of 1 Watt. 0,1 Lux relates to red light (750 nm) under the same conditions. Examples of light intensities: - sunny summer day outside: about 100000 lux - covered sky in summer: about 20000 lux - dimmed winter day: about 3000 lux - good street lamp: about 40 lux - night with full moon: about 0,25 lux. Good lightening helps to avoid accidents. For work there are regulations which demands 100 to 250 lux and 1000 lux for precision work. The brightness/ intensity of light depend also on the distance of the lightsource. The intensity decreases quadratically with the distance.
Reference to reality	Ride into a tunnel (without seeing the end), distance from a lamp (street lamp, desk lamp etc.)

(Experiment tunnel)

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Worksheets and Impulses

The investigation of the functional relation and the hypothesis should be stimulated by an everyday life impulse, which should lead to the interdisciplinary background and a talk about it.

Before starting the experiment students should deal with the material:

- What can be changed?
- When we change one quantity, which quantity changes simultaneously?
- Which relation do you presume?

In principle there is the <u>general task</u> above every experiment

Describe the relation between quantity … and quantity ….   Verify: Does the relation confirm your presumption?   Describe the special features of the relation.

Follow:

- Impulses for every experiment
- Worksheet for the experiment *Electric car* as an example
- General worksheet for to fill in

Impulses (to copy - see next pages)

Impulse **Electric Car**

Imagine you sit in this car. The car

1. starts at a traffic light.
2. goes around a corner.
3. keeps going straight on a long highway.

Describe the different movements of the car.
Talk about it in your group.

Impulse **Drop**

Sure sometimes you saw a tap dropping, perhaps in your kitchen or in the bathroom.

What's your opinion? Is it important to turn off the tap or to repair it?

Estimate the number of drops per hour, per day? How many litres will that be?

Is there a relation between the number of drops and the volume of the dropped water?

Talk about all the questions in your group.

Impulse **Free fall**

In leisure parks or fairs you can find a special attraction, the "Freefall Tower". This is a slim tower of iron bars about 50 m high. The people are first brought up to the top and then dropped down. Do you have experiences with it? What did you feel?

The falling distance could be different at different towers. What would be the difference between a long and a short fall?

Discuss this in your group. Find many differences.

Source: www.pxelquelle.de ID99300, fotograf: anjume

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Impulse — Cylinder

Medicine and food supplements like vitamins etc. are sold as pills, liquid or powder. Powder medicine and other food are often packed in cylindrical tins.
Why are cylindrical tins so useful?
Discuss this in your group.

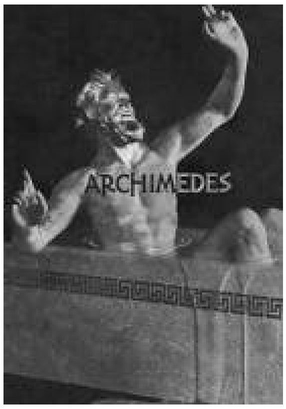
Suppose there are two tins of same height but different radius of the base. What about their volume in case the radius of the base of one tin is twice as big as the other.
Discuss this in the group and mark:

- o the volume is the same
- o the volume is 1.5 times bigger
- o the volume is double
- o the volume is four times bigger.

Impulse — Globe

Do you know the story about Archimedes (287-212 BC) in the bath.
What happened?
Could you say something about the volume of the displaced water?
Are there differences when small or bigger persons go into the bath?

Discuss this in your group.



source: Tamara Schuh 1C2, www.lgh.lu

On the table you can see a measuring jug you can fill with water; and globes of different radii.

Think about:

- Which globe will displace most water?
- Look at the globe and assume: Does the globe with double radius will displace double volume of water?

Discuss this in your group.

Impulse **lever 2**

In playgrounds sometimes you can find seesaws. Sure you seesawed before. Perhaps you tried (together with a partner) to balance the seesaw.
Imagine there are friends to seesaw with you. Jack is heavier than Alex.
Assume: Who has to sit nearer to the axis for a balanced position?

Discuss this in your group.

In the experiment we built a simple seesaw. You can fix the force meter so that the level is balanced.
Assume:
Do you need more or less force the nearer the force measure is to the axis?

Discuss this in your group.

Impulse **Tunnel**

Imagine you go into a tunnel and you cannot see the end.
How does the brightness (the intensity of light) changes while going into it?

Discuss this in your group,

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Worksheet *Electric Car*

Equipment

On the table you can see
- an electric carriage
- a measure tape
- stopping watches

Take some minutes to get to know the car. Let it go. Which of the movements is comparable with the car's movement?
- starting at a traffic light.
- going around a corner.
- keep going straight on a long highway.

Let the car go 20 cm, 40 cm etc. What can you say about the times needed? Think about a relation.

Talk about all the questions in your group.

Performance

1. Let the car go 70 cm and measure the needed time.

Fill into the empty box:

70 cm →

2. Now, let the car go 100 cm. Which time corresponds to it?

Fill in:

→

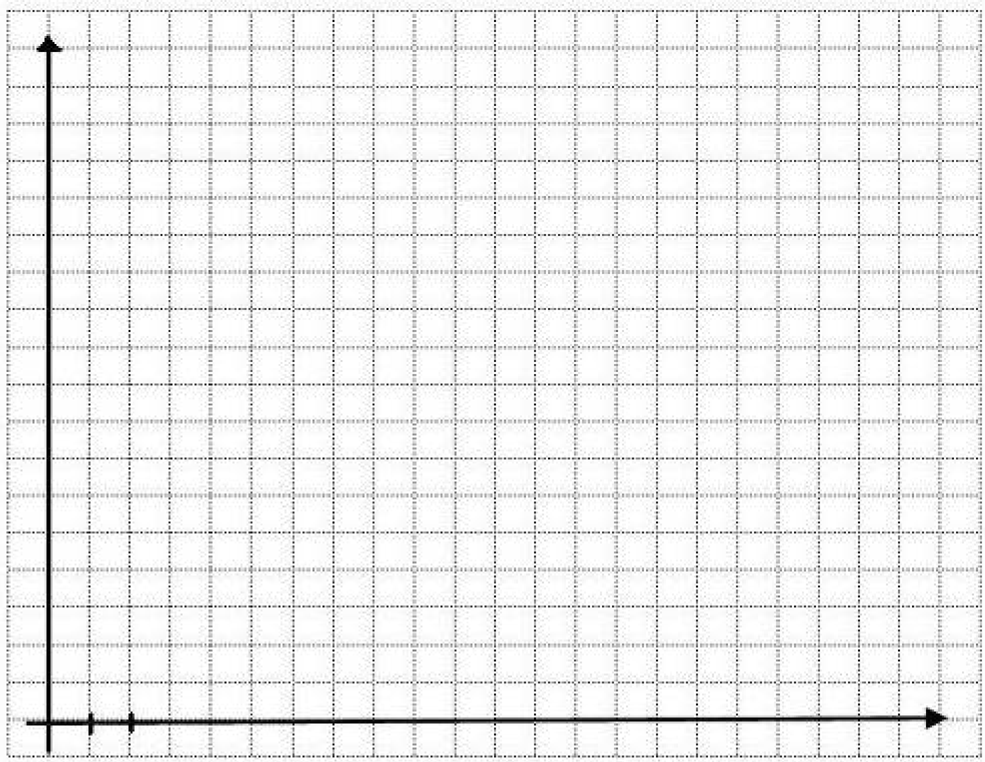
3. Put corresponding values into the table.

First fill the dependent quantities into the first column (with units).

4. Consider the table. Are there any relations? Which?
   Note all you have discovered.

5. Plot the values from the table into the coordinate-system.

Inscribe the axis:
Inscribe at the x-axis the time.
Inscribe at the y-axis the distance.



6. Consider the graph.
Describe it.

Describe it in dependence of the values of the quantities.

7. Investigate the graph.

a) Note. Which time does the car need for 130 cm, 180 cm, 260 cm?
Fill the values into the table. Compare.

b) Mark in the graph the change from 60 cm to 120 cm with a big line. Then mark the corresponding change at the y-axis. Make a big line.

c) Now take another colour and mark the change from 30 cm to 90 cm and the corresponding change at the y-axis.

d) Compare the changes in b) and c). Describe the difference.

8. Now look at the graph again. Which distance does the car drive in 6 s?

Which distance does it go in 60 s (1 minute)?

Which distance does it go in 1 hour?
Compare: Is it quicker as a pedestrian?

9. Document the results of this station clearly arranged on a poster.

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Worksheet

<u>Equipment</u>

On the table you can see

<u>Performance</u>

1. Measure

and measure __________

Fill in:

2. Measure more

Which belongs to it. Fill in:

3. Put corresponding values into the table.

First fill the dependent dimensions into the first column (with units).

4. Consider the table. Are there any relations? Which?

Note all you discover.

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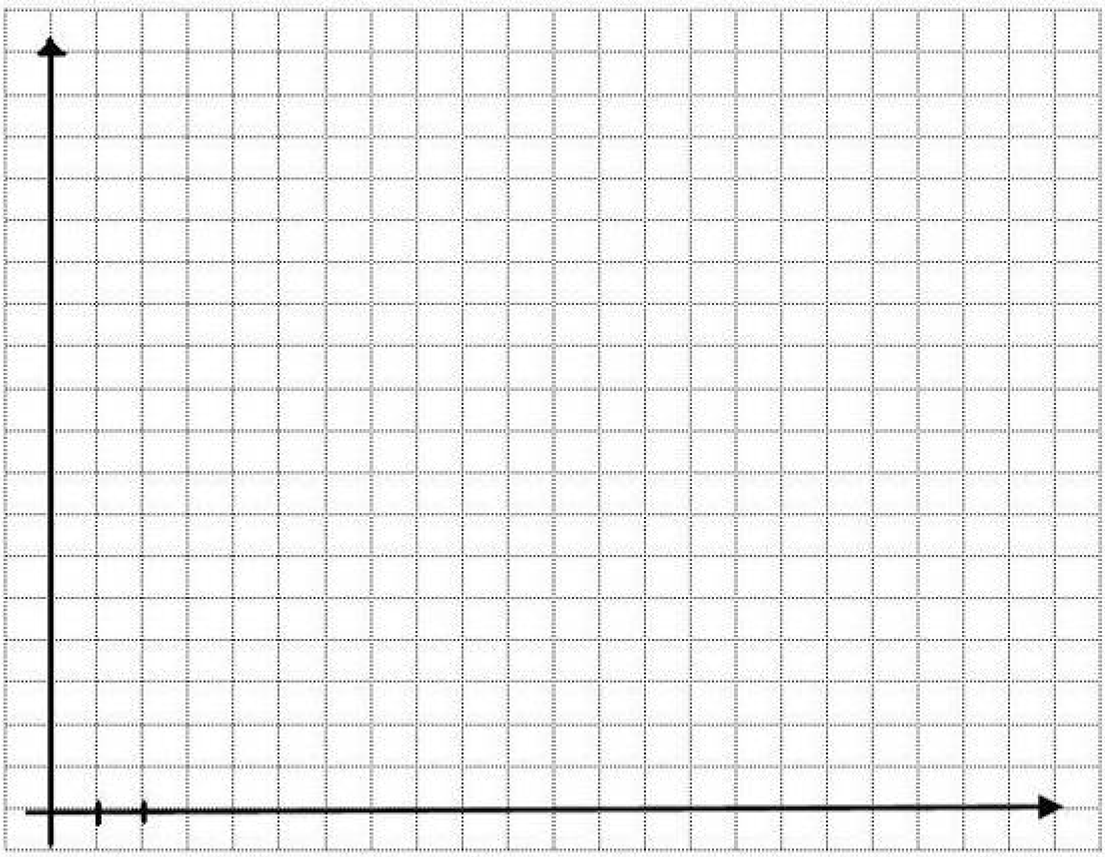
5. Plot the values from the table into the coordinate-system.

Inscribe the axis:
Inscribe at the x-axis the quantity you changed.
Inscribe at the y-axis the quantity which changed in reaction.



6. Consider the graph.
Describe it.

Describe it in dependence of the values of the quantities.

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7. Investigate the graph.

a) Note. Which value of (x-axis) correspond to (y-axis) and to (y-axis)?

Fill the values into the table. Compare.

b) Mark in the graph the change from to at the x-axis. Make a big line. Then mark the corresponding change of at the y-axis. Make a big line.

c) Now take another colour and mark the change from to at the x-axis and the corresponding change at the y-axis.

d) Compare the changes in b) and c). Describe the difference.

8. Document the results of this station clearly arranged on a poster.