Education and Culture
Lifelong learning programme
COMENIUS

Background

General didactic background

Starting point is an interdisciplinary approach with science. Students shall experience Mathematics reasonable, significant and interesting by extra-mathematical references; learning in contexts shall contribute to an intuitive mathematic understanding. By means of scientific contexts and methods the often watched gap between formal maths and authentic experience shall be closed on the one hand and versatility of mathematic terms shall be experienced on the other hand.

Scientific contents offer the possibility for realistic teaching. Concrete physical or biological correlations may initiate mathematical activities and lead to authentic experiences. Mathematic themes and methods are apprehended in reasonable contexts; reality of pupils may be extended by mathematical understanding. Various realistic references lead to different models and may so contribute to distinction of conceptual attributes and of different models. The variety of scientific phenomena allows open tasks and so self-dependent development of mathematics. Mathematical items e. g. concept of function may be experienced as modelling tools. The coherences of meanings and the differing attributes may be detected within various realistic references.

The idea of teaching implementation

Basic idea of the teaching implementation is that the pupils experience the concept of function by experimental activities, realistic references and scientific contexts. Experiments are chosen because:

- the experimental steps correspond to the aspects of the concept of function
- these aspects are realized in a practical way
- the aspects are experienced authentically.

Various presentation levels and changes in between are initiated through various modelling activities. Due to reference to reality and concrete quantities of the experiment the often neglected and less skilled competences in regard to the interpretation of graphs will be incited (see detailed description → Literature). There will be especially an occasion to recognize functional relations and to discuss them; the aspect of covariation may be experienced in an authentic manner.

The incitation for the experimental activities is given at different stations. By realistic impulses the pupils will be remembered of their own experiences in every-day-life and discussions about changing characteristics of dimensions and framing of hypotheses will be initiated. The verification of the hypothesis motivates doing the experiment which will lead to a functional relation. This relation is normally noted in a table and will then be examined graphically. Importance is attached to the verbal debate and the relation to every-day-life (at least as conclusion). Final presentations of each working group in a classroom-session have proved to be good for this.

Mathematical background

The concept of function is one of the most important however even one of the most difficult mathematical concepts. Many examinations have shown that pupils often have limited comprehension of it. Function is often seen as “something with x and y” or “something that can be pictured graphically”. In a graph of a function there is rather seen the course of the line than the functional dependence between two quantities.
In fact there is a danger to reduce the treatment of functions in teaching above all to the drawing of graphs out of equations. The concept of function however is much more complex: To know the concept of function means to be familiar with its textual ideas, its various levels of representation and the change between. The complexity of the concept of function has been subject of a lot of investigations during the past decades. Important works to this topic have been drawn up by DeMorois & Tall, Stoye, Fischer & Malle, and Swan, who identified different forms of representation, the change between and cognitive levels.
Following three different aspects of the concept of function are distinguished in order to recapitulate:

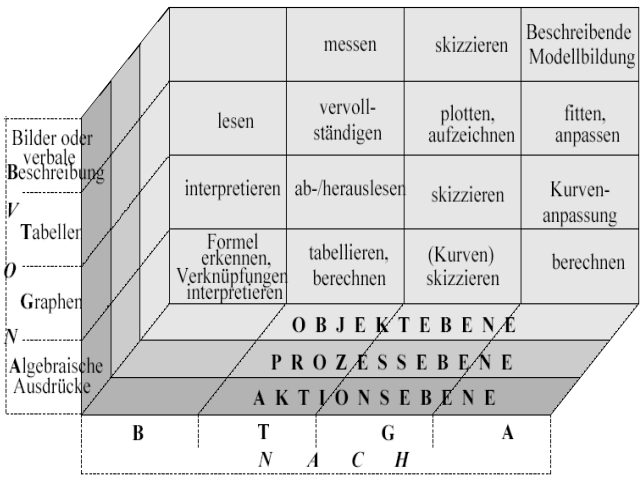
Aspect of correspondence (action):
each element x of a set X is assigned to exactly one element y of a set Y. In a simple case we can regard only one element x. Or we regard all x of X in order or continuously.

Aspect of covariation (process):
If x is changed the corresponding y will change, too. Thereby we can change x discretely or let the set X pass through continuously.

Object aspect:
To comprehend a function as object means to comprehend the function as a whole, which means to be familiar with aspects like simple and continuous correspondence, discrete and continuous covariation in all forms of representation, possible changes and ways of changes.

Representating aspects are
situation (pictures, descriptions), table, graph, algebraic term.

The concept of function is most extensively comprehended in the “Haus des funktionalen Denkens“/ “house of the functional thinking” developed by Höfer (see figure). All aspects of the concept of function and all possibilities of a change are considered and explained there. However it also allows differentiation between various possibilities of one and the same translation, e. g. whether the graphic implementation of a function is made point wise or considering dynamic aspects.



House of the functional thinking (Thilo Höfer 2006)

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