Idea: Thilo Höfer,
Staufer Gymnasium Waiblingen, Germany

Teaching Material

Suggested Lesson Plan

If the students are not familiar with the "Lifeguard" problem (Cf. ScienceMath unit "Fermat meets Pythagoras" by the same author), then it should be introduced directly with the help of the worksheet available (item worksheet WS 1) or in a different form (e.g. teacher's presentation). If the Lifeguard-problem is known, instruction can take place in the form of group work with the help of the following worksheet "Fermat's Principle". Here, each group should have an experimental setup at their disposal.
As a variation, each group can be given a variety of clear liquids to investigate, in order to determine the speed of light for these materials.

Material needed

Each group needs one set of material. The following is needed for the experiment (cf. picture):

- 1 glass container (minimum length 40 cm and minimum height 30 cm),
- water or other clear liquid,
- 1 laser pointer,
- 1 snail or comparable accessory,
- stand,
- measuring tape

Worksheets

Fermat's Principle

Light does not travel at the same speed in different materials. The speed of light in air for example is about 300000km/s. In glass it is only about 200000km/s. In addition, light does not always behave like a perfect lifeguard, e.g. a ray of light always travels from A to B by the path that is the shortest for the light. This behaviour of light is called Fermat's Principle, named after the scientist who formulated it first.

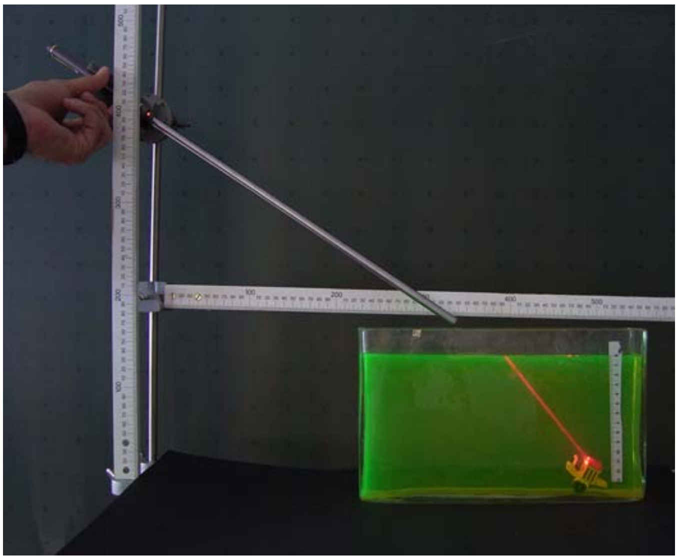

figure 1: snail in water

Tasks:
A fresh water snail would like some light on its shell. To achieve this, the ray of a laser pointer outside the water is directed exactly at the snail's shell (see figure 1).

- **a)** If one measures the arrangement in figure 1 and transfers it to a Cartesian system of co-ordinates, the laser pointer appears at point L $(0/y_L)$, the line describing the surface of water follows a straight line g with the equation y=b, and the top of the snail's shell is point $S(x_S/y_S)$. The laser pointer's ray of light hits the water at $Q(x_Q / b)$ (all measured in cm). Get the missing values from the experimental set-up and enter Q, L, g and S in a co-ordinate system.
- **b)** Determine the speed of light of the ray in water, using the value for the speed of light in air mentioned above.

possible solution b)

e.g. a= 26 (level of laser pointer from water line), b= 11 (distance shell to water line), x= x_Q =41 (distance laser-pointer to dip point on water line).

c-x=50-x_Q= 9
v_1 =300.000 km/s = 30.000.000.000 cm/s

The following applies in air: $t_1 = \sqrt{\frac{a^2 + x^2}{v_1}}$

The following applies in water: $t_2 = \sqrt{\frac{b^2 + (c-x)^2}{v_2}}$

Thus: $$t(x) = \frac{\sqrt{a^2+x^2}}{v_1} + \frac{\sqrt{b^2+(c-x)^2}}{v_2} = \frac{1}{v_1}\cdot\sqrt{a^2+x^2} + \frac{1}{v_2}\cdot\sqrt{b^2+(c-x)^2}$$

In principle, all values are known except v_2. First, however, one should start from the assumption that x is not given and v_2 is known. One would like to calculate the point at which the ray enters the water. In this function, the x for the fixed constants in this experiment (except x) is chosen in such a way that t becomes minimal (Fermat's Principle). Thus, that x is sought, for which the time function t(x) has a low point, so that x definitely has to be chosen in such a way, that the derivative is 0.

$$t'(x) = \frac{1}{v_1}\cdot\frac{1}{2}(a^2+x^2)^{-0,5}\cdot 2x + \frac{1}{v_2}\frac{1}{2}\cdot(b^2+(c-x)^2)^{-0,5}\cdot(2c-2x)$$
$$= \frac{1}{v_1}\cdot x\cdot(a^2+x^2)^{-0,5} + \frac{1}{v_2}\cdot(c-x)\cdot(b^2+(c-x)^2)^{-0,5}$$

This term must be equals 0.

$$0 = \frac{1}{v_1}\cdot x\cdot(a^2+x^2)^{-0,5} + \frac{1}{v_2}\cdot(c-x)\cdot(b^2+(c-x)^2)^{-0,5}$$

Only now does one "turn the tables": As x is known, one knows for which x this derivative takes the value 0. However, one doesn't know the respective v_2.
That's why, one resolves v_2 and enters all the other values.

$$v_2 = \frac{(c-x)\cdot(b^2+(c-x)^2)^{-0,5}}{\frac{1}{v_1}\cdot x\cdot(a^2+x^2)^{-0,5}}$$
$$= \frac{9\cdot(11^2+9^2)^{-0,5}}{\frac{1}{30.000.000.000}\cdot 41\cdot(26^2+41^2)^{-0,5}}$$
$$\approx 2{,}249\cdot 10^{10}$$

=> $2{,}25\ 10^{10}$ cm/s = $2{,}25\ 10^{8}$ m/s = 225.000 km/s

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