Education and Culture
Lifelong learning programme
COMENIUS

Background

General didactical background

The basis of this sequence is an interdisciplinary approach with sciences especially with Physics. The pupils shall experience Mathematics in an appropriate, interesting and important way by the means of extra-mathematical references. Learning in interrelations shall contribute to an intuitive mathematic understanding. With the aid of scientific contexts and methods the gap between formal mathematics and authentic experience shall be closed and on the other hand the variety of mathematic items shall be experienced.
Scientific contents open a chance for realistic teaching. Concrete physical or biological phenomena can stimulate modelling processes and can lead to authentic experiences. Mathematical themes and methods are learnt in meaningful contexts; reality can be expanded by adding a mathematical view. Different real contexts lead to different models and can show various meanings of concepts and models. The richness of scientific phenomena allows open tasks and hands-on approaches to mathematics. Mathematical concepts, like the concept of variable, can be experienced as a modelling tool. In various authentic contexts the multifaceted meanings can be explored.

Physical didactical background

In optics instruction usually concentrates first on the radiation model. Here, the direction of light rays is studied and predicted. In order to do this, the phenomena of light reflection and light refraction are observed first.
While the prediction of a rays' path regarding light reflection poses no problem in class, due to the realisation that "The angle of incidence equals the angle of reflection.", light refraction poses a problem: If one does not only want to retrace it, one at least has to have a knowledge of the sinus, in order to predict the direction of the rays in refraction with the help of the equation: sin a1 / sin a2 = n2/n1 and given refraction figures n1 and n2. However, even if this equation has been introduced, there remains an "after taste", since two phenomena, that both are based of the same physical principle – Fermat's principle – are taught together without any connection.

Fermat's Principle in Class

To start off with, it sounds very simple: A ray of light follows a path between two points in such a way that it uses the least possible time (see Vogel, p. 174). This principle is not changed, even if one imposes the condition of a detour via a mirror: "The reflected ray of light follows the shortest path, which leads via the mirror from A to B" (ibd. p. 173).
If the medium and thereby the speed of the light's diffusion on its way from A to B is changed, as is the case in the light refraction phenomenon, the time-wise shortest path between two points A and B is no longer the geometrically shortest connection. While trying to work out the temporally shortest path, one hits a minimising problem with a target function, which consists of a sum of two root terms. In a physics class with 16 to 18 year olds, one can assume that they possess the mathematical means to analytically evaluate such a minimising problem. Therefore, Fermat's principle will be used immediately after its introduction to train, respectively to deepen, their mathematical capabilities in the field of differential calculus. Simultaneously, the speed of light in water (with the knowledge of the speed of light in air) can be determined. That way, a direct connection of physics and mathematics is achieved, in which both sciences exist side by side on an equal footing.

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