Background

In early Physics teaching when dealing with the field of optics main focus is usually on the model of light as a beam. By using it the ways beams of light move are comprehended and predicted by taking advantage of the already regarded phenomena of light reflection and refraction. Because of the realisation “angle of incidence equals angle of reflexion”, predicting the ways beams of light move when talking about reflection does not cause any difficulties when teaching it. Light refraction, however, appears to cause a problem: If you don’t only want to comprehend but predict the ways beams of light move, knowledge regarding sine is required. Refraction of light can be predicted by the equation $\sin \alpha_1 / \sin \alpha_2 = n_2 / n_1$ and given refraction indices n_1 and n_2.
However, even with introducing this equation there is still a certain ”bad taste” about it, as there are now 2 phenomena deriving from the same physical principle -Fermat’s principle- which has been taught without being connected in any way.

Teaching Fermat’s Principle

First of all it sounds very easy: A beam of light moving from one point to another always chooses the quickest possible way (acc. Vogel, p.174). This principle does not change even if conditions are changed and the beam of light has to take a “detour” via a mirror: “The reflected ray of light follows the shortest path, which leads via the mirror from A to B” (ibid. p.173).
If the medium and therefore the speed for the spreading of light is changed on the way from A to B -as it is the case for refraction- the quickest way (considering time) between the two points A and B is not the shortest way (considering distance) anymore. When trying to calculate the quickest way you are faced with a minimising problem containing a target function that consists of the sum of two root terms.
When teaching Physics in a class of 13-15 year-old pupils you cannot assume that they have the mathematical means for analytical evaluation for such a minimising problem at their disposition. They will fail calculating the solution when they have formulated the target function at the latest if not before. That is why as an alternative to differential calculus the solution to the minimising problem needs to be carried out by the graphs of the accompanying function. In order to not make this reliable on the pupils drawing skills on the one hand or limited time in class on the other hand (as a detailed calculation of the objective function would be necessary) the opportunity to use a function plotter (PC, graphing calculator) presents itself. In doing so the pupils will find solutions which may consist of function classes they do not know yet.

Does it even make sense for the pupils to investigate functions with the aid of CAS or GTR that they were not able to investigate without these?

In order to answer this question we have to remember that functions cannot only be pictured by terms but also by situations, tables and graphs etc. (see Beckmann, Leuders & Prediger, and more). Even a chart is an illustration of a function. It is true that the pupils may not examine the function on the basis of its term. They may, however -as a computer does automatically- change the term into a chart. The function, being displayed as “graph”, could then be investigated regarding its low point. You would, of course, have to accept

certain inaccuracy in meter-reading which would not occur when using a computer (except for a minimal inaccuracy within the approximation of the low point).
This way the pupils could indeed investigate the function by themselves. The computer would only help them by minimising the sources of error. On the contrary, it is very reasonable to confront the pupils with function terms which they could not evaluate mathematically without using a computer. The Pedagogic Centre of Rheinland-Pfalz (Pädagogisches Zentrum Rheinland-Pfalz) found out that the abilities of the pupils concerning functions are quite one-sided. This one-sidedness is accredited to the "predominance of presentation of functions by algebraic terms". Resulting from this predominance a "drastic confinement of the functional connections" can for example be seen when functions of Middle-school classification are used almost all the time (linear, quadratic…functions). On the other hand an "overemphasis on inner-mathematical, static and formal abstract approaches" is regarded as a cause (see PZ 1990, p.9). In order to change the one-sidedness within training of functional thinking, it may make sense to choose the method mentioned above from time to time before turning to fast technical help.

This project has been funded with support from the European Commission. This publication reflects the views only of the author(s), and the Commission cannot be held responsible for any use which may be made of the information contained therein.