Education and Culture
Lifelong learning programme
COMENIUS

Background

General didactic Background

Starting point is an interdisciplinary approach with science. Students shall experience Mathematics as reasonable, significant and interesting by extra-mathematical references; learning in contexts shall contribute to an intuitive mathematic understanding. By means of scientific contexts and methods the often watched gap between formal maths and authentic experience shall be closed on the one hand and versatility of mathematic terms shall be experienced on the other hand.
Scientific contents offer the possibility for realistic teaching. Concrete physical or biological correlations may initiate mathematic activities and lead to authentic experiences. Mathematic contents and methods are apprehended in reasonable contexts; reality of pupils may be extended by mathematical understanding. Various realistic references lead to different models and therefore may contribute to a distinction of conceptual attributes and of different models. The variety of scientific phenomena allows open terms of references and so self-dependent development of mathematics. Mathematic items e. g. concept of function may be experienced as modelling tools. The coherences of meanings and the differing attributes may be detected within various realistic references.

Regarding the theme “Special Points in a Triangle”, a more global view and interconnections can be reached by experiments, in which the interception point of the medians of a triangle is one of many examples of the centre of gravity in bodies.

The didactical Implementation – the Idea

The basic idea behind the implementation in class is to allow students to experience the concept of the centre of gravity via an assessment of its position in a real situation, respectively, in scientific contexts and through experimental activities. The situations are presented at various stations. By passing through various stations, different aspects of the topic are worked out. The interception of the medians of a triangle will be recognized as a special case and will be integrated into the topic “centre of gravity in bodies and areas”.

The Subject Background

The Mathematics-Natural Science Concept: Centre of Gravity

One topic in maths instruction is “Special points in the triangle”. This includes, among other things, discovering that the medians of a triangle intersect in a common point. That is remarkable but that is not all. Looking at it in a natural-science context, it opens up special opportunities for a multifaceted, applicable and realistic elaboration, because the intersection is at the same time the centre of gravity of a triangle.

Definition
A body’s centre of gravity is the intersection of its lines of gravity.

Explanation of Terms

A line of gravity is a line where the force of gravity acts.

The weight F_g of a body is its force of gravity. If we imagine the body as being composed of many mass particles, its weight is the result of all partial weights which act on each mass particle. It can be determined by the product of total mass m of the body and the gravitational acceleration $g = 9.81\frac{kgm}{s^2}$, i.e. $F_g = mg$.

If the sites x_i of all mass particles m_i of a body are known, the point of gravity is

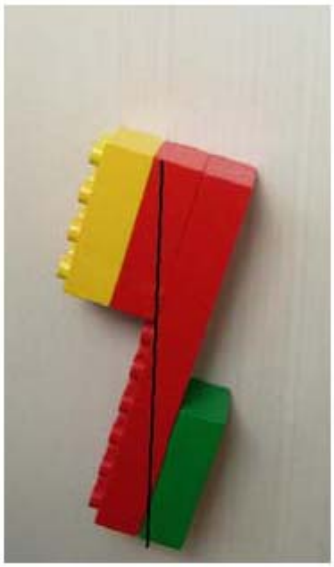
$$\vec{s} = \frac{1}{m_1 + \ldots + m_k} \sum_{i=1}^{k} m_i \overrightarrow{x_i}$$

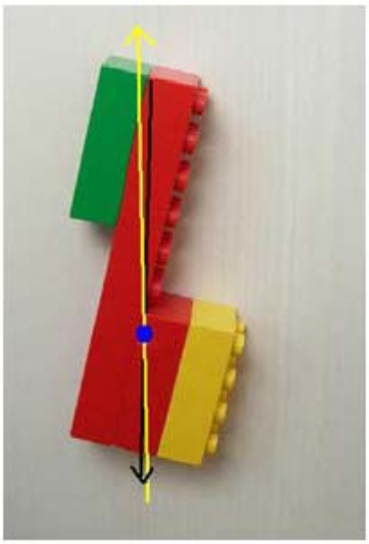
The centre of gravity can be visualized as the point of origin of a weight. Thus, if a weight of equal size but opposite force impacts at that point, the body is in balance.

Methods to determine the centre of gravity result from this definition.

1. Suspension Method:

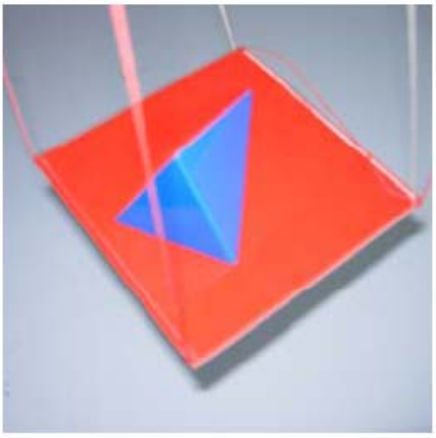
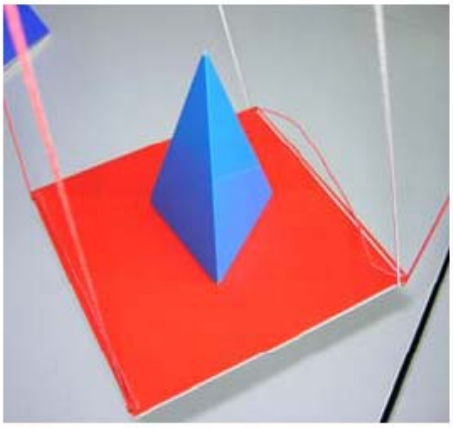
A body is hung up in two positions, and in each positiion the respective lines of gravity are determined (picture). Then, the point of intersection will be constructed.





2. Weighing Method:

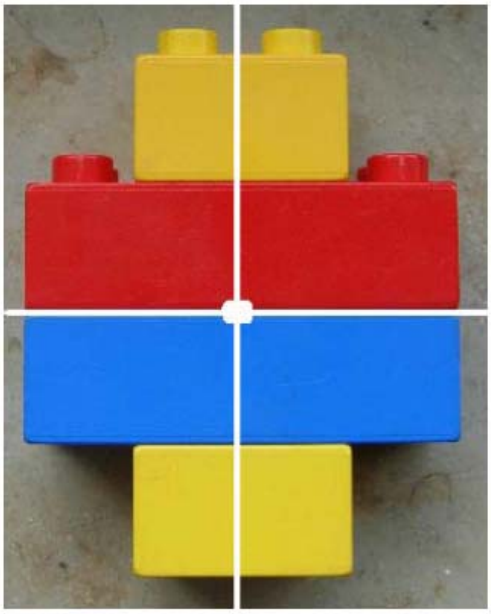
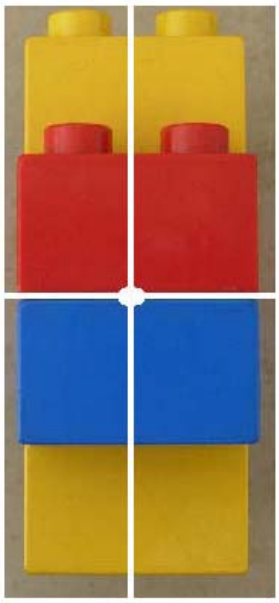
A body is placed in two different positions on a platform, which is hung on a string and put in balance. The lines of gravity are worked out and their point of intersection is determined (picture).

3. Supporting Force Measuring Method:

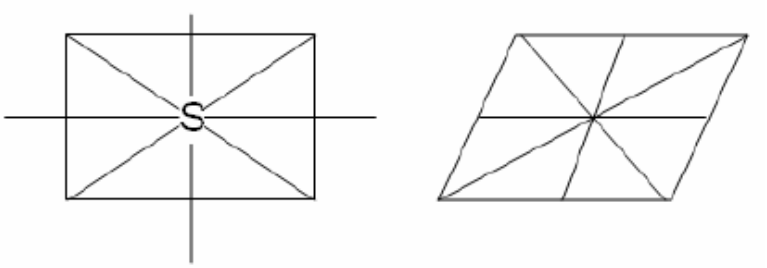
(for large, non-suspendable bodies)
The determination of the lines of gravity is done by measuring the supporting forces in two different body positions.

An important result that can be gained from experiments made by the suspension method is:
In homogenous bodies are all symmetry lines gravity lines
(in the two-dimensional graphs below some symmetry lines are indicated; the position of the point of gravity is indicated; it is in the centre of the body).

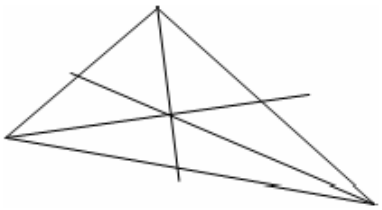



These considerations can be transferred to the determination of the centres of gravity in planes and straight lines.

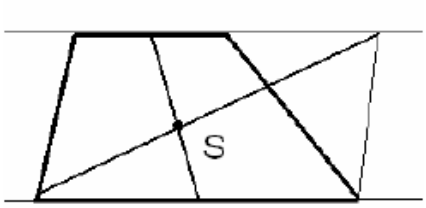
The centre of gravity of:
Parallelograms/rhomboids, rectangles: point of intersection of the medians, as well as the diagonals



Triangles: point of intersection of the medians

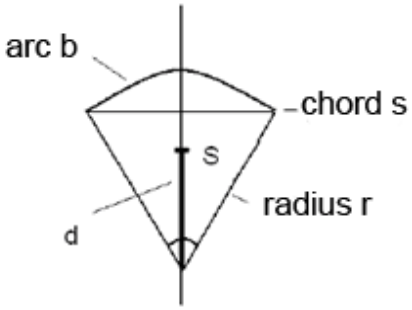


Trapeze: point of intersection of the medians of the two parallel sides with the diagonal of the complementary parallelogram



Segment of a circle with radius r, arc length b and the chord length s:

On centre line at distance of $d = \frac{2}{3} \cdot \frac{rs}{b}$



Note:
The centre of gravity in composed planes results from the momentum theorem on the centres of gravity of partial planes. The momentum theorem postulates the distance of the centre of gravity from an axis (e.g. y-axis) as the sum of the products of the partial areas and distances of their centres of gravity from the axis (Cf. also materials).

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