The **ScienceMath-**project: **Arc Length**
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Education and Culture
Lifelong learning programme
COMENIUS

Teaching Material

The formula

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx \qquad (1)$$

answers the question: **What is the length of a plane curve which is given by the equation y = f(x)?** In order to make the task simpler, let us assume that f(x) and f'(x) are continuous functions in the interval [a, b].
Usually we don't know the answer without knowing this formula.

However, the **case of a circle** is an exception. That is the reason one can find very often this example (a circle) in the textbooks. It is quite suitable to calculate a quantity using the two independent ways. One fourth of a circle circumference is therefore calculated using primary school formula $o = 2\pi r$ and the second option, by using the quoted formula (1), which is derived only during very advanced pre university course or in the first year of the university course.

I suggest that the circle is not the only case. Therefore we make the two types of calculation. The parabola is also an example. This curve is connecting physics and mathematics

See Teaching Module
PARABOLA BETWEEN MATHEMATICS AND PHYSICS THE CASE OF HORIZONTAL LAUNCH.

The **parabola** equation is:

$$y = \frac{g}{2v_0^2}x^2$$

The y axis is oriented downwards. We are going to calculate the length of the trajectory. The initial velocity is 5,0 m/s, the acceleration of gravity is 10,0 m/s^2 and certainly, we neglect the air resistance. The initial position is the (0, 0) point.
Assume that the hits the floor after 2,0 s after it has been launched. Its coordinates are:

$$x = v_0 t \qquad \text{and} \qquad y = \frac{gt^2}{2}$$

Therefore, x = 10 m and y = 20 m. Certainly, the trajectory is longer than the distance between (0, 0) and (10, 20).

We can calculate that distance as follows:

$$y = \frac{g}{2v_0^2}x^2 \qquad \text{and} \qquad y' = \frac{g}{v_0^2}x$$

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$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx =$$

$$= \int_a^b \sqrt{1 + \left[\frac{g}{v_0^2} x\right]^2}\, dx$$

We write: $a = \frac{g}{v_0^2}$, therefore

$$L = \frac{1}{a} \int_0^{10} \sqrt{a^2 + x^2}\, dx =$$

$$= \frac{1}{2a} \left[x\sqrt{x^2 + a^2} + a^2 \ln(x + \sqrt{x^2 + a^2}) \right]_0^{10}$$

= 23,23

The length of the trajectory is 23,23 m.

It is easy to calculate the magnitude of the instantaneous velocity in the case of horizontal launch.
The magnitude of the instantaneous velocity is:

$$v = \sqrt{{v_0}^2 + (gt)^2}$$

If we divide the launch in a large number of very short time intervals we assume that the velocity within a time interval is a constant one. We multiple each velocity with the time interval and the result is the displacement during this time interval. We just sum up all the displacements and the length of the trajectory is calculated. Although such a calculation (for one displacement) is a simple one, we must calculate many “steps” which corresponds to many time intervals. That is the reason to leave this task to Excel.
We write in a **spreadsheet** (see next page):

and we pull down to row 206. There time will be 2,00 s.
The parabola length is 23,23, calculated using Excel.

Certainly we could simply use Excel instead of integration. In this case one would use the equation:

$$d\ell = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

But this is only a replacement of integration. However it is equal in one way to the physics’ derivation, because $dx = v_x dt$ and $dy = v_y dt$…

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Spreadsheet:

	A	B	C	D
1	time interval		initial velocity	
2	0,01	s	5	m/s
3				
4				
5	t	v	delta x	L
6	0	=$C$2	0	0
7	=A6+$A$2	=SQRT($C$2^2+(10*A7)^2)	=((B7+B6)/2)*$A$2	=D6+C7

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